I

6701 - structural Dynamics and Earthquake Engineering

Unit - I - Theory of vibration

Difference b/w static & dynamic loading

Engineering Mechanics deals with behaviour of body with the state of rest or motion under the action of forces

Static — body is rest

Dynamics — body is motion

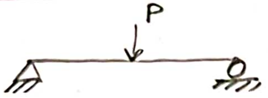
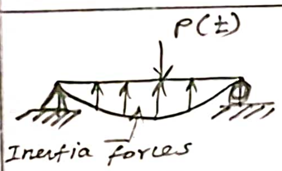
- Kinematics — study of motion (without considering all forces)
- kinetic (considering all the forces)

$R = \sqrt{\Sigma H^2 + \Sigma V^2}$

Newton's Law

1 → It state of rest or motion of the rigid body is unaltered unless it is acted upon by the external forces.

2 → applied force is directly proportional to the rate of change of momentum $\boxed{F \propto a}$

Static load	dynamic load
	
Load is constant with respect to time	Load is varying with respect to time
It has only one response ie, Displacement	Three responses (i) Displacement (x) (ii) velocity ($\dot{x}$) (iii) acceleration ($\ddot{x}$)
It has only one solution	Infinite number of solutions. (based on time dependent)
Static Analysis is easy	Dynamic analysis is more complex & more time consuming
Response can be calculated by the principles of static equilibrium $\Sigma H = 0$; $\Sigma V = 0$; $\Sigma M = 0$	Total responses are calculated by including inertia forces along with the static equilibrium $F = Ma$

3 → Every action has an equal to an opposite reaction.

Unit - I Introduction to Dynamics

Vibration → motion of the structure repeats its after a given interval of time of motion.

Vibration Destructive

- * structure/machine → failure/fatigue.
- * human beings → discomfort & noise created (trouble) to human beings
- * Instrument panels → malfunction interference with reading

Vibration constructive

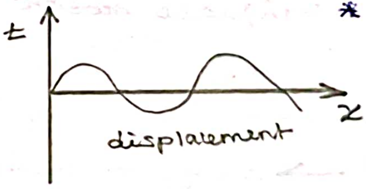
- * Musical Instruments
- * vibrating equipment (concrete compactor, vibratory conveyors, hoppers, sieve, pile driving etc)
- * vibration associated with casting & welding
- * vibration table for industrial product testing.

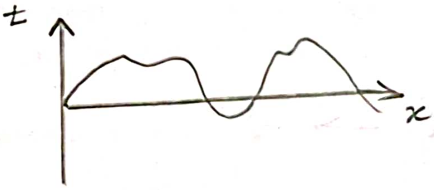
Periodic loading

→ Simple harmonic motion Same number of cycle.

~~which having~~ time variation for

* Rotating machinery in building

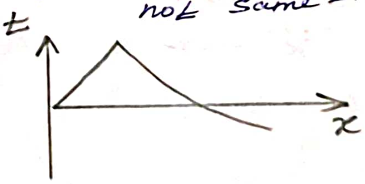




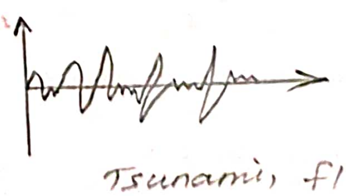
Propeller forces stain of ship

Non - Periodic loading

→ It does not same time vibration



Bomb blast loading



Earthquake load on water tank

Tsunami, flood etc.

Causes of Dynamic effects in Structure

- Natural Sources
- Manmade Sources

① **Initial conditions** → velocity & displacement produce dynamic affect

Example:- Lift moving up & down initial velocity formed suddenly stopped then cabin begins to vibrate up & down.

② **Applied forces** → Some times vibration in the system is produced due to the application of external forces.

Example:-

a) Building subjected to a bomb blast or wind forces

(b) Machine foundation.

③ **Support motions** → subjected to vibration due to the influence of support motions.

Example:- Earthquake motion

Basic Definitions

Displacement (x) → change of position of body (or) motion of particle from one place to another place

Velocity ($\dot{x}$) → The rate of change of displacement is known as velocity.

$$\boxed{V = \frac{dx}{dt}}$$

Acceleration ($\ddot{x}$) → The rate of change of velocity is called as acceleration.

$$\boxed{a = \frac{dx^2}{dt} \text{ or } \frac{dV}{dt}}$$

Mass (m) → It is a property of body to resist external force. unit → kg $\frac{\text{weight}}{\text{acceleration}}$

$$\boxed{m = \frac{\text{Wt. of body}}{\text{acceleration due to Gravity}} = \frac{W}{g}}$$

Stiffness (k) → The force required to produce unit deformation.

$$\boxed{k = \frac{\text{load}}{\text{Deflection}} = \frac{W}{\Delta}} \quad N/m$$

Natural period (T) → Time required to complete one cycle of free vibration. unit sec.

Natural frequency (f) → The number of cycles per unit time.

unit → rad/sec (or) Hertz.

$$f = \frac{\overset{\text{one cycle}}{1}}{T} \quad \text{Time period} \quad \text{CPS}$$

frequency

one cycle

$$\omega_n = \frac{2\pi}{T} \qquad \frac{1}{T} = f$$

$$\omega_n = 2\pi f$$

$$\therefore \boxed{f = \frac{\omega_n}{2\pi}}$$

angular frequency

cycle

Amplitude → The maximum displacement (or) deformation of a vibrating system from its mean position is called Amplitude.

Inertia → The tendency of a body to resist the change of state is called as inertia.

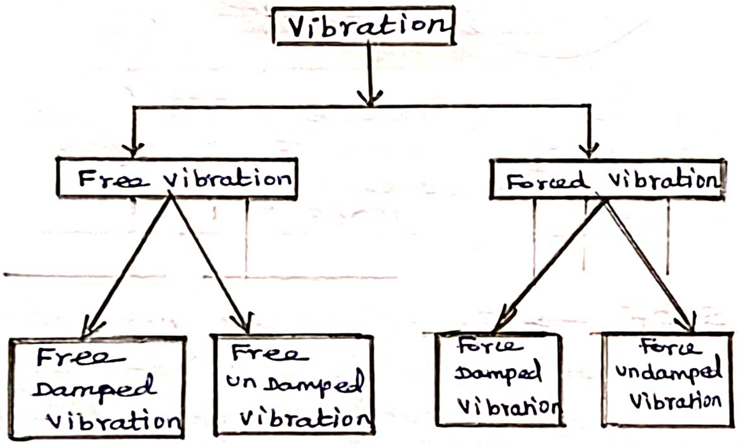
Impulse → The huge amount of load is applied for a short period of time.

Damping $\xrightarrow[\text{is the}]{\text{Unit N/m/sec.}}$ resistance to the motion of vibrating body. The vibrations associated with this resistance are known as damped vibration.

Sources of dynamic loads in Civil Engg. Practices

- * Structures subjected to alternating forces caused by oscillating machinery.
  Example : Machine foundation
- * Moving load on bridges.
- * Structures subjected to **suddenly applied forces** such as blast load, wind load.
  Example : Tall building
  Antena Tower, chimney.
- * Movement of foundation of the structure due to earthquake.

Types of vibration



Vibration → The motion of the structure repeat its after a given interval of time of motion is called vibration.

Free vibration → Vibration that exists without the presence of external force.

Example: Vibration that continues after wind force has stopped or earthquake has stop in a tall building, water tank, chimney, cooling tower etc.,

Forced Vibration → Vibration that exists with the **presence of external force.**

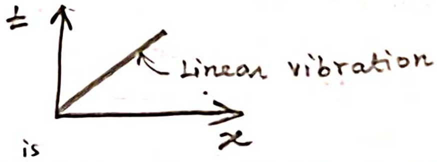
Damped Vibration → When a damper or damping element is attached to the vibrating system, is known as damped vibration.

- * stiffness & forces taken into consideration
- * Damping force considered.

Undamped vibration → When a damper (or) damping element is not attached to the vibratory system is known as undamped vibration.

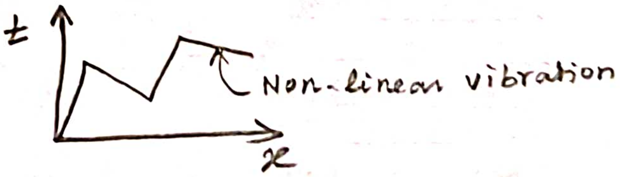
- * stiffness & forces taken into consideration
- * Damping force is not considered.

Linear vibration → Basic components (spring, mass & damper) of a vibrating system behave in a linear manner, the resulting vibrations caused are known as Linear Vibration.



* It is obey the law of superposition.

Non-Linear Vibration → The basic components of a vibratory system behave in a non linear manner, the resulting vibration is called non-linear vibration.



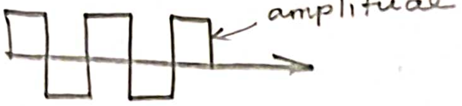
* It does not follow the law of superposition.

Deterministic vibrations → The amount of excitation (force & motion) acting on a vibrating system is completely known precisely, the resulting vibrations are called Deterministic vibrations.

* Velocity ($\dot{x}$), displacement (x), acceleration ($\ddot{x}$) are exactly known

Random vibrations →

The amplitude are constant through the periodic value



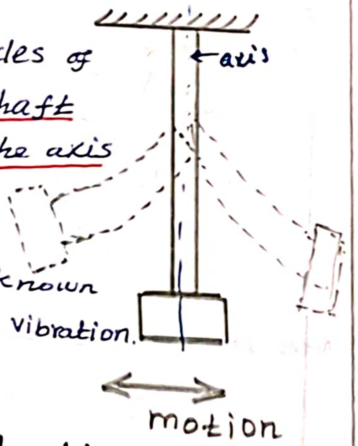
* When the amount of excitation (force & motion) is not completely known, the resulting vibrations are known as Non-deterministic (or) Random vibration.

* It is used to analyse the earthquake excitation of building & structure.

* Force are not exactly known

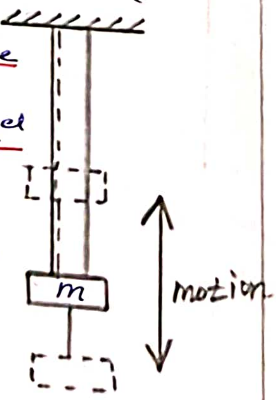
Transverse Vibration →

* When the particles of the body (or) shaft move ⊥r to the axis of the shaft, the vibrations created are known as transverse vibration.



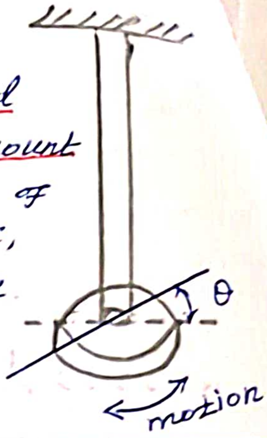
Longitudinal Vibration →

* If the mass of the vibratory system moves up & down ‖el to the axis of the shaft, the vibrations created as longitudinal vibrations.



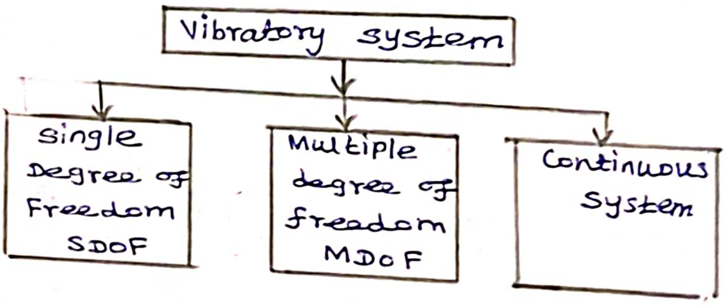
Torsional Vibration →

* If the shaft gets alternately twisted & untwisted on account of vibratory motion of the suspended disc, Such vibrations are called torsional Vibration.



Degree of freedom

Degree of freedom → The minimum number of independent co-ordinates required to indicate the position/motion of a system at any time instant is known as degree of freedom



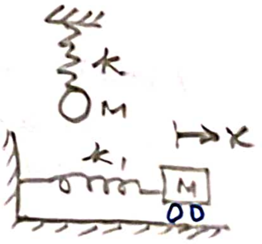
SDOF → If the system where only one co-ordinate is sufficient to define the position/motion of a system at any time instant is called SDOF.

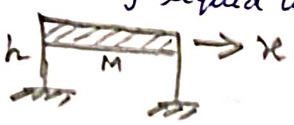
Example :

(1) one mass one spring

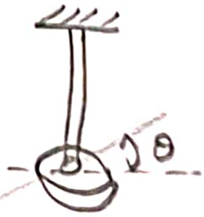
(2) simple pendulum

(3) Vibrating liquid column

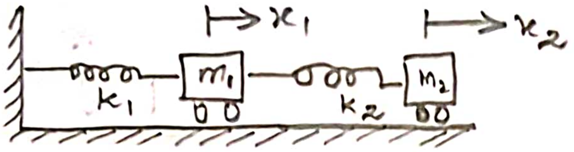


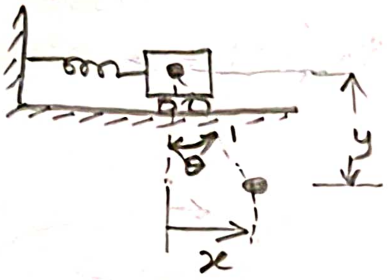


4) Torsional system with one disc & connected to a torsional spring



MDOF → If more than one independent co-ordinate is required to completely specify the position or geometry of different masses of the system at any instant of time, it is called MDOF



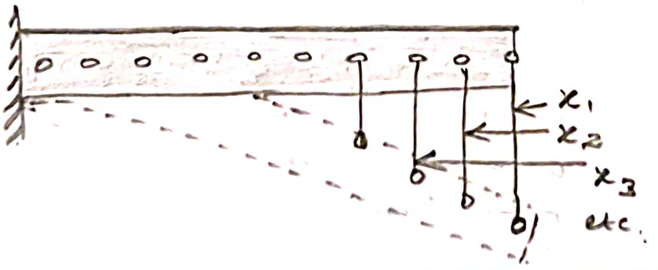


Continuous system → If the mass of a system may be considered to have infinite degree of freedom, it is known as continuous or distributed system.

Example:-

1) A cantilever or any other beam where the mass & the restoring force are distributed.

2) A shaft subjected to many steps (or) subjected to continuous twists.



Response → is defined as the magnitude & distribution of the resulting forces and displacements in a system due to vibration.

Free Response → motion due to initial condition

Forced Response → When the motion is due to applied forces, it is known as forced response.

Resonance → the frequency of external force is equal to or matches with one of the natural frequencies of the vibrating system, the amplitude of vibration becomes excessively large. This phenomenon is called resonance.

$$f_{ext.force} = nat.\ f$$

Amplitude very large

Consequences of Vibration

- * over stressing & Collapse of Structure
- * cracking & other damage requiring repair
- * Damage to saftey related equipment
- * Impaired performance of equipment or delicate apparatus
- * Adverse human response
- * Fatigue fracture.

Vibration Control — Design of Structure

3 steps for design of structure.

- * Identifying the dynamic loads interms of frequency & amplitude or measured variation with time
- * Analysing the response of the Structure → to obtain dynamic deflections, Stress, frequencies & acceleration.
- * checking the calculated or measured performance against specified criteria to ensure that there are no adverse consequences of vibration

Types of Damping

Damping → is the resistance to the motion of vibrating body.

Damped vibration → The vibrations associated with this resistance are known as damped vibration.

- * The vibrating energy of the system is gradually reduced (or) The amplitude of vibration is slowly decreased.
- * Unit → N/m/sec.

Damping force → The resisting force that shall be applied on a system to prevent from vibration or oscillation

Types of Damping

Solid material

Structural Damping → due to the internal molecular friction of the material of the structure.

* Due to the loss of energy associated with the slippage of structural connection.

Viscous Damping → When a system is made to vibrate in a surrounding medium or under the control of highly viscous fluid (Petrol, Kerosine, oil, water)

vibration in a fluid

Damping force $F = \frac{\mu A \dot{x}}{t}$

$\therefore$ $\boxed{F = c . \dot{x}}$

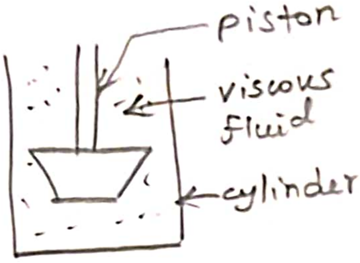
μ → co-efficient of absolute velocity of fluid

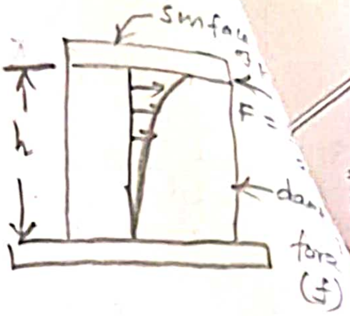
t → thickness of plate

A → surface area of plate

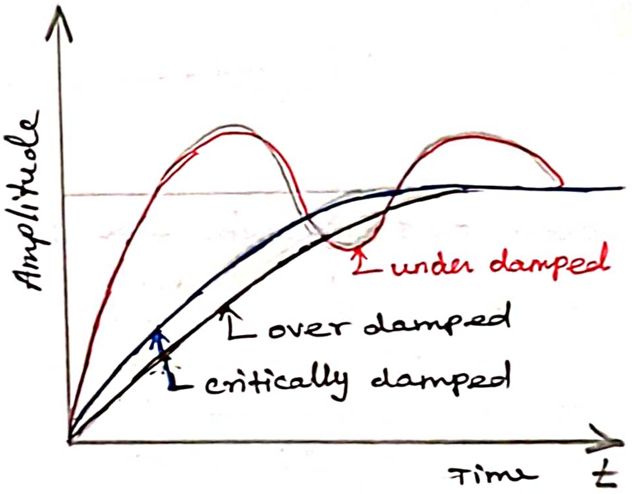
$\dot{x}$ → velocity

c → damping co-efficient





[illegible] Damping →



Coulomb damping → The energy absorbed to sliding ~~the~~ friction.

- * It is also called dry friction.
- * Friction is developed by the relative motion of two surfaces that slide against each other is a source of energy dissipation.

$\boxed{F = \mu . N}$

μ → co-efficient of static friction

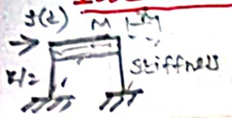
F → frictional force

N → Normal reaction

Active Damping $\xrightarrow{\text{it refers}}$ energy dissipation from the system by external loads (controlled action)

Passive Damping → Energy dissipation with in the structure by damping devices such as isolated structural joints and supports or structural member internal elements.

Idealisation of structure as SDOF system



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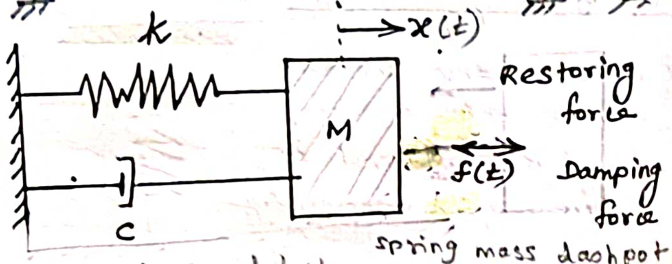


spring mass dashpot ↗ system

(Vibration) a device for damping shock

According to Newton's Second Law

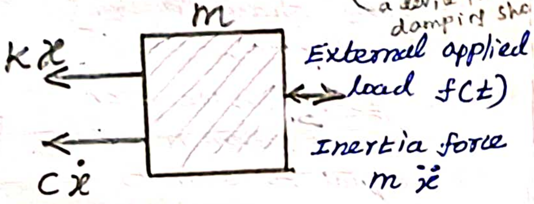
$F \propto a$

$F = ma$

$F = m\ddot{x}$

inertia force ③

SDOF system



Free body Diagram

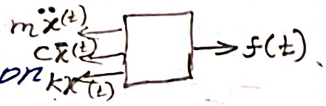
x → lateral displacement

$\dot{x}$ → velocity

$\ddot{x}$ → acceleration

m → Lumped mass

k → lateral stiffness



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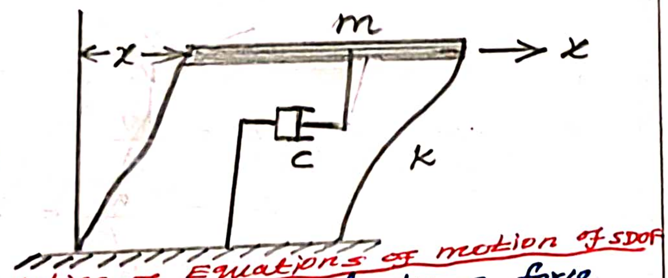


Formulation of Equations of motion of SDOF SDOF subjected to a force

Restoring force acting opposite to the motion producing

$$\boxed{\text{Restoring force} = k \cdot x} \quad \text{— ①}$$

~~spring force~~ / elastic force

* Damping force also acts opposite to the motion (ie, resisting force)

* Assumed to be proportional to velocity of the moving mass equal to $c \cdot \dot{x}$

where c → damping co-efficient

$\dot{x}$ → velocity

$$\boxed{\text{Damping force} = c \cdot \dot{x}} \quad \text{— ②}$$

* Inertia force acting on the mass is the product of mass and absolute acceleration & acts opposite to the motion.

$$\text{Inertia force} = m \cdot \frac{d^2x}{dt^2}$$

$$\boxed{\text{Inertia force} = m \cdot \ddot{x}} \quad \text{— ③}$$

$\ddot{x}$ → absolute acceleration of the mass

Equilibrium of forces gives the equation of motion of the system is using D'-Alembert principle

$$\boxed{\text{Inertia force} + \text{Damping force} + \text{Restoring (or) Elastic force} = \text{Applied force}}$$

~~spring force~~

$$F_I + F_D + F_E = f(t)$$

$$\boxed{m \cdot \ddot{x} + c \cdot \dot{x} + kx = f(t)}$$

For forced damped vibration

For Undamped free vibration,

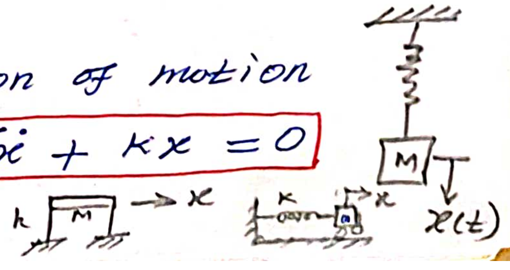
(free)

damping co-efficient $c = 0$

External force $f(t) = 0$

∴ Equation of motion

$$\boxed{m\ddot{x} + kx = 0}$$



For damped Free vibration

external force $f(t) = 0$

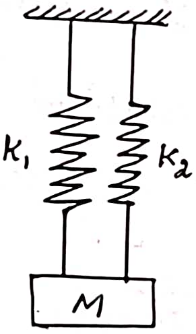
∴ The equation of motion

$$\boxed{m\ddot{x} + c\dot{x} + kx = 0}$$

Equivalent stiffness of spring combination

- * When a system have more than one spring, the spring may be connected in series or parallel or both. in the case.
- * Some times it may be connected inclined position, it can also be replaced by difference skill by the same stiffness as they all shown the same stiffness as a whole.

Springs in parallel



- * The spring in a system subjected to a common deflection & the total load supported is the sum of the individual loads shared by each spring.

$$\Delta = \Delta_1 = \Delta_2$$

where, $\Delta, \Delta_1, \Delta_2 \rightarrow$ static deflection of the spring

$$W = W_1 + W_2$$

$$\text{stiffness}(k) = \frac{\text{load}}{\text{deflection}} = \frac{W}{\Delta}$$

$$W = k \cdot \Delta$$

$$\therefore W_1 = k_1 \cdot \Delta_1$$

$$W_2 = k_2 \cdot \Delta_2$$

$$\therefore k\Delta = k_1\Delta_1 + k_2\Delta_2 \quad (\because \Delta_1 = \Delta_2 = \Delta)$$

$$\therefore k_e\Delta = k_1\Delta + k_2\Delta$$

$$\therefore k_e = \frac{(k_1 + k_2)\Delta}{\Delta}$$

$$\therefore \boxed{k_e = k_1 + k_2}$$

$k_e \rightarrow$ equivalent stiffness of the system

Springs in series

- * consider two linear springs of stiffness k_1 & k_2 arranged in series. as shown in fig.
- * When the springs are connected in series, if they share a common load,
- * the total deflection of the system must be equal to sum of the deflection of the individual springs.
- * Let the individual static deflections in springs of stiffness k_1 & k_2 under the same axial load W be Δ_1 & Δ_2 respectively.

$$\therefore \text{Total Deflection} \quad \Delta = \Delta_1 + \Delta_2$$

$$W = W_1 = W_2$$

$$\therefore k = \frac{W}{\Delta}$$

$$\therefore \Delta = \frac{W}{k}$$

$$\Delta_1 = \frac{W_1}{k_1} = \frac{W}{k_1}$$

$$\Delta_2 = \frac{W_2}{k_2} = \frac{W}{k_2}$$

$$\therefore \frac{W}{k_e} = \frac{W}{k_1} + \frac{W}{k_2}$$

$$\therefore \boxed{\frac{1}{k_e} = \frac{1}{k_1} + \frac{1}{k_2}}$$

Methods to Derive the equation of motion

1. Simple Harmonic motion method (SHM Method)
2. Newton's Method
3. Energy method
4. Rayleigh's Method
5. 'D'. Alembert's Principle

1. Simple Harmonic motion Method (SHM Method)

* It is one of the forms of Periodic motion.

* Harmonic motion represented interms of ~~cosine~~ circular of sine and cosine functions.

* For a particle in a rectilinear motion, if its acceleration is always proportional to the distance of the particle from a fixed point on the path & is directed towards the fixed Point.

$$\ddot{x} \propto -x$$

rectilinear displacement

$$\ddot{x} = -\omega_n^2 x$$

constant of proportionality

$$\boxed{\ddot{x} + \omega_n^2 x = 0}$$

$x \rightarrow$ rectilinear displacement

$\dot{x} = \frac{dx}{dt} \rightarrow$ velocity of the particle

$\ddot{x} = \frac{dx^2}{dt^2} \rightarrow$ acceleration of the motion.

$(-)^{ive}$ sign $\xrightarrow{\text{indicate}}$ direction of motion

2. Newton's Second Law of motion

* Newton's 2^{nd} Law of motion state that the rate of change of momentum. is proportional to the impressed forces and takes place in the direction.

* consider a spring-mass system, is assumed to move only along the vertical direction.

* It has only one degree of freedom, because its motion is a single co-ordinate 'x'.

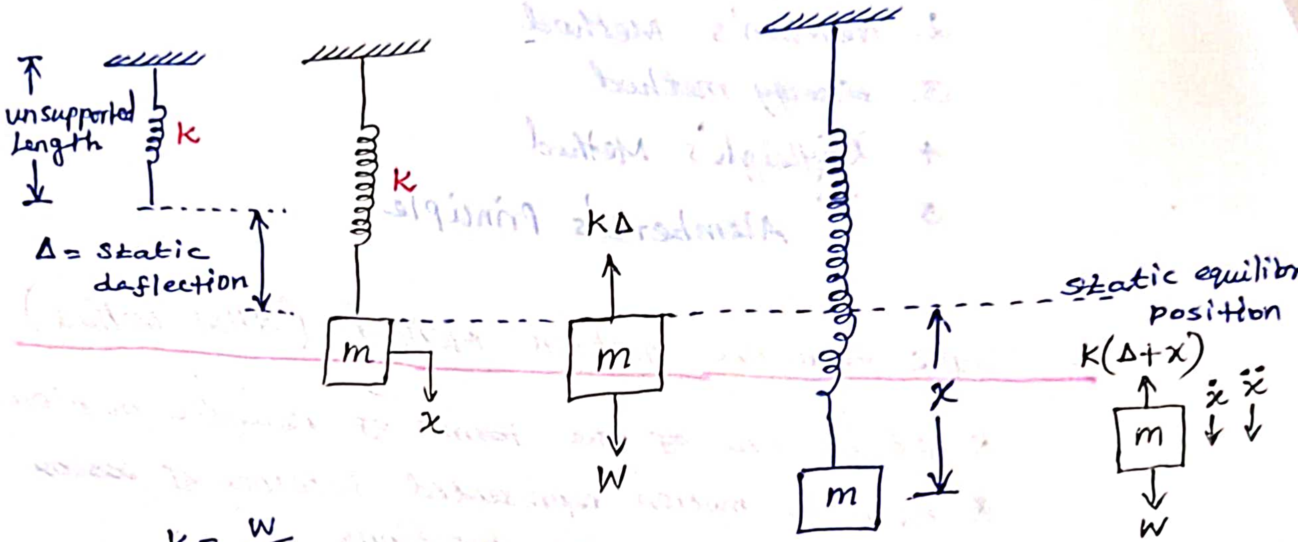
Stiffness → load required to produce unit deformation

* Spring factor $\boxed{k = \frac{W}{\Delta}}$

W → load

Δ → static deflection

* spring elongates or displaces from its equilibrium Position Vertica downwards. This Position is called equilibrium Position.



$K = \frac{W}{\Delta}$

Spring mass system & FBD
↙ ↙ ↘
Free Body Diagram

$\therefore \boxed{W = k.\Delta}$

* Load 'W' pulled down a little, by some force & then the pulling force is removed. The load 'W' will continue to execute vibrations up & down is called Free Vibration

$$\text{Restoring force in } x\text{-direction} = W - k(\Delta + x)$$

$\boxed{W = k.\Delta}$

$$= W - k\Delta - kx$$

$$= k.\cancel{\Delta} - k\cancel{\Delta} - kx$$

~~Restoring force~~ ~~$m\ddot{x}$~~

$$\boxed{\text{Restoring force} = -kx}$$

According to Newton's Law,

Change of momentum is proportional to the impressed force

$$m\ddot{x} = -kx$$

$$\boxed{m\ddot{x} + kx = 0}$$

unit

Stiffness (k) → N/m

Mass (m) → kg

Natural frequency or Angular frequency (ω_n) → rad/sec

$\div$ by m, $\ddot{x} + \frac{k}{m}x = 0$

$$\boxed{\ddot{x} + \omega_n^2 x = 0}$$

$\omega_n = \sqrt{k/m}$

$\omega_n^2 = \frac{k}{m}$

$$\omega_n = \sqrt{\frac{k}{m}} = \sqrt{\frac{N/m}{kg}} = \sqrt{\frac{\frac{\cancel{kg}.\cancel{m}/s^2}{\cancel{m}}}{\cancel{kg}}} = \sqrt{\frac{1}{s^2}} = \frac{1}{s}$$

3. Energy Method

- * Total sum of energy is constant at all time.
- * For undamped system, → no friction (or) damping force
- * Total energy of the system is ~~[illegible]~~ Partly potential & Partly kinetic.
- * Law of conservation energy,

Total energy = constant

K.E + PE = constant

Kinetic energy + potential energy = constant

- * The time ~~rate~~ rate of change of total energy will be zero

(I) ——

$$\boxed{\therefore \frac{d}{dt}(KE+PE)=0} \text{ ——— } ①$$

$$KE = \frac{1}{2}mv^2 = \frac{1}{2}m\dot{x}^2 \quad \Big| \quad PE = \frac{1}{2}kx^2$$

(II) ——

$$① \Rightarrow \therefore \frac{d}{dt}\left(\frac{1}{2}m\dot{x}^2 + \frac{1}{2}kx^2\right)=0$$

$$\cancel{\frac{1}{2}m \cdot 2\dot{x} \cdot \ddot{x}}$$

$$\left(\frac{1}{\cancel{2}}m\cancel{2}\cdot\dot{x}\cdot\ddot{x} + \frac{1}{\cancel{2}}\cdot k\cdot\cancel{2}x\cdot\dot{x}\right)=0$$

$$m\dot{x}\ddot{x} + kx\cdot\dot{x} = 0$$

$$\dot{x}(m\ddot{x}+kx)=0$$

÷ by $\dot{x}$

$$\therefore \boxed{m\ddot{x}+kx=0}$$

4. Rayleigh's Method → mostly used to determine ω_n

- * It is assumed that the
- * Max. KE at the equilibrium position equal to the Max. Potential energy at extreme position
- * motion is assumed.

$$x = A\sin\omega_n t$$

displacement → x; time → t

x is max so $\sin\omega_n t = 1$

$$\therefore \boxed{x_{max} = A}$$

differentiate w.r. to time ,

Velocity $\dot{x} = \omega_n A \cos\omega_n t$

Velocity is only maximum when $\boxed{\cos\omega_n t = 1}$

$$\therefore \boxed{\dot{x}_{max} = \omega_n A}$$

Max. KE at equilibrium position $= \frac{1}{2} m v^2$

$$= \frac{1}{2} m \dot{x}_{max}^2$$

$$= \frac{1}{2} m (\omega_n A)^2$$

$$\boxed{KE_{max} = \frac{1}{2} m \omega_n^2 A^2} \text{ —— (I)}$$

Max. PE at extreme position $= \frac{1}{2} K \cdot x_{max}^2$

$$\boxed{PE_{max} = \frac{1}{2} K \cdot A^2} \text{ —— (II)}$$

Equating eqn (I) & (II)

$$\cancel{\frac{1}{2}} m \omega_n^2 \cancel{A^2} = \cancel{\frac{1}{2}} K \cdot \cancel{A^2}$$

$$\omega_n^2 = \frac{K}{m}$$

$$\therefore \boxed{\omega_n = \sqrt{K/m}}$$

ω_n → Natural frequency rad/sec.

m → mass in kg

K → stiffness in N/m

D - Alembert's Principle

According to Newton's second Law

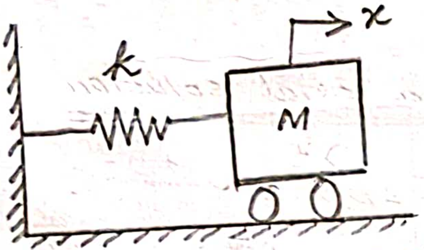
$$F \propto a$$

$$F = ma$$

$$F - ma = 0$$

① The above equation is in the form of equation of motion of force equilibrium in which the sum of a number of force terms equals zero

③



spring Mass system

Free Body Diagram (or) Dynamic equilibrium

- * Imaginary force (or) Inertia force = ma = $m \cdot \ddot{x}$

②

- * Imaginary force ma were applied to the system in the direction of opposite to the acceleration.
- * This system could be considered to be in equilibrium under the action of real force 'F' & the imaginary force ma
- * The position of equilibrium is called dynamic equilibrium.
- * 'D'-Alembert's principle state that ~~a system~~ a system may be in dynamic equilibrium by adding to the external forces, an imaginary.

Equilibrium equation = $\Sigma F_x = 0$

Spring (or) Restoring force = $k \cdot x$

$\therefore \Sigma F_x = 0$

Inertia force + Spring (or) Restoring force = Applied force

There is no applied force

$$m\ddot{x} + k \cdot x = 0$$

÷ by m

$$\ddot{x} + \frac{k}{m} x = 0$$

$\omega_n^2 = k/m$

$\omega_n = \sqrt{\frac{k}{m}}$

$$\ddot{x} + \omega_n^2 x = 0$$

$$\omega_n^2 = k/m$$

Natural (Angular) frequency $\therefore$ $\boxed{\omega_n = \sqrt{\frac{k}{m}}}$

$$\boxed{\ddot{x} + \omega_n^2 x = 0}$$

Formulate the equation and to find out the response of an undamped free vibration of SDOF

Solution:-

The equation of motion for undamped free vibration is

$$m\ddot{x} + kx = 0 \text{ ——— } ①$$

Assuming a trial solution

displacement $x = A \sin \omega t$

velocity $\dot{x} = \frac{dx}{dt} = \omega A \cos \omega t$

acceleration $\ddot{x} = \frac{d^2x}{dt^2} = -A\omega^2 \sin \omega t$

$① \Rightarrow m\ddot{x} + kx = 0$

$m(-A\omega^2 \sin \omega t) + k(A \sin \omega t) = 0$

forces, which is commonly known as the inertia force.

$$A\sin\omega t\left(-m\omega^2+k\right)=0$$ zero

$$-m\omega^2+k=0$$

$$k=m\omega^2$$

$$\omega^2=\frac{k}{m}$$

$$\therefore \boxed{\omega=\sqrt{\frac{k}{m}}}$$

Where, ω → angular frequency

k → stiffness

m → mass

Natural frequency $\boxed{(f)=\frac{1}{T}}$ $\frac{\text{one cycle}}{\text{sec}}$ cps

Where,

T → Time period (Time taken for 1 oscillation)

one cycle = 2π

$$\therefore \omega=\frac{2\pi}{T} \qquad \frac{1}{T}=f$$

$$\boxed{\omega=2\pi f} \text{ rad/sec}$$

Assuming a trial solution

Hyperbolic function

$$x=A\sin\omega t+B\cos\omega t \text{ ——— ②}$$

$$\dot{x}=A\omega\cos\omega t-B\omega\sin\omega t$$

The constant A & B can be found out from the initial boundary conditions.

(i) At $t=0$; $x=x_0$

$$② \Rightarrow x=A\sin\omega t+B\cos\omega t$$

put $t=0$

$$\therefore x_0=0+B$$

$$\therefore \boxed{x_0=B}$$

$$\dot{x}=A\omega\cos\omega t-B\omega\sin\omega t$$

At, $t=0$; $\dot{x}=\dot{x}_0$

$$\dot{x}_0=A\omega-0$$

$$\dot{x}_0=A\omega$$

$$\therefore \boxed{A=\frac{\dot{x}_0}{\omega}}$$

Substituting the A & B values in eqn ② $x=A\sin\omega t+B\cos\omega t$

$$\boxed{x=\frac{\dot{x}_0}{\omega}\sin\omega t+x_0\cos\omega t}$$

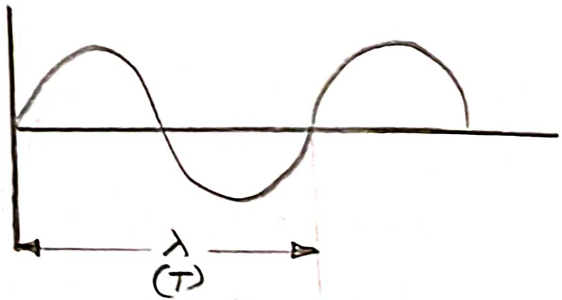
Assuming a trial solution

SP22

$$x=Ae^{\lambda t} \text{ ——— ⓐ}$$

$$\dot{x}=A\lambda.e^{\lambda t} \text{ ——— ⓑ}$$

$$\ddot{x}=A\lambda^2e^{\lambda t} \text{ ——— ⓒ}$$



one cycle

The general Equation of motion is

$$m\ddot{x}+kx=0 \text{ ——— ①}$$

substitute the eqn ⓐ, ⓑ & ⓒ in eqn ①

$$mA\lambda^2e^{\lambda t}+k.Ae^{\lambda t}=0$$

÷ m

$$A\lambda^2e^{\lambda t}+\frac{k}{m}Ae^{\lambda t}=0$$

$$A\lambda^2e^{\lambda t}+\omega^2Ae^{\lambda t}=0$$

$$\omega=\sqrt{\frac{k}{m}}$$
$$\omega^2=\frac{k}{m}$$

$\div Ae^{\lambda t}$

$$\lambda^2 + \omega^2 = 0$$

$$\lambda^2 = -\omega^2$$

$$\boxed{\lambda = \pm i\omega}$$

$\sqrt{-1} = i$

$x = Ae^{\lambda t}$ put $\lambda = \pm i\omega$

So the solution is $x = Ae^{\lambda t}$

$$x = A_1 e^{i\omega t} + A_2 e^{-i\omega t}$$

(or)

Formula $e^{ix} = \cos x + i\sin x$

$$x = A_1(\cos\omega t + i\sin\omega t) + A_2(\cos\omega t - i\sin\omega t)$$

By rearranging

$$x = c_1\cos\omega t + c_2\sin\omega t \quad \text{———} ②$$

where, c_1 & c_2 → constants.

c_1 & c_2 are determined from the initial condition

(i) $t = 0$; $x = x_0$

substituting $t = 0$; $x = x_0$ in eqn ②

② ⇒ $x = c_1\cos\omega t + c_2\sin\omega t$

Put $t = 0$; $x = x_0$

$$x_0 = c_1 + 0$$

$$\therefore \boxed{c_1 = x_0}$$

Differentiate w.r.t eqn ②

$$\dot{x} = -c_1\omega\sin\omega t + c_2\omega\cos\omega t$$

substitute $t = 0$; $\dot{x} = \dot{x}_0$

$$\dot{x}_0 = 0 + c_2\omega$$

$$\dot{x}_0 = c_2\omega$$

$$\boxed{c_2 = \frac{\dot{x}_0}{\omega}}$$

c_1 & c_2 Values in equation ②

$$\boxed{x = x_0\cos\omega t + \frac{\dot{x}_0}{\omega}\sin\omega t} \checkmark$$

Assuming a trial solution $x = A\cos(\omega_n t - \xi)$

Assume The general solution is

Comparing with general solution $x_0 = A\cos\xi$

$$\frac{\dot{x}_0}{\omega} = A\sin\xi$$

By squaring and adding x_0 & $\frac{\dot{x}_0}{\omega}$ we get,

$$x_0^2 + \frac{\dot{x}_0^2}{\omega^2} = A^2\cos^2\xi + A^2\sin^2\xi$$

$$= A^2(\cos^2\xi + \sin^2\xi) \leftarrow = 1$$

$$x_0^2 + \frac{\dot{x}_0^2}{\omega^2} = A^2(1)$$

$$\therefore \text{Amplitude } (A) = \sqrt{x_0^2 + \frac{\dot{x}_0^2}{\omega^2}}$$

Amplitude $\boxed{A = \sqrt{x_0^2 + \left(\frac{\dot{x}_0}{\omega}\right)^2}}$

Similarly dividing $\left(\dot{x}_0/\omega\right)$ by x_0

We get $$\left(\frac{\dot{x}_0}{\omega}\right) \Big/ x_0 = \frac{A\sin\xi}{A\cos\xi}$$

$$\left(\frac{\dot{x}_0}{\omega}\right) \Big/ x_0 = \tan\xi$$

$$\boxed{\tan \xi = \frac{\dot{x}_0}{x_0 \omega}}$$

$\therefore$ Phase angle $\boxed{\xi = \tan^{-1} \frac{\dot{x}_0}{x_0 \omega}}$

Problem

1. Find the natmal frequency of the free vibration system subjected to a weight (W) 15 N is vertically suspended by a spring of stiffness is 2 N/mm.

Given Data :-

Weight (W) = 15 N

$$\text{Mass } (m) = \frac{W}{g} = \frac{15}{9.81}$$

$$\boxed{m = 1.529 \text{ kg}}$$

$$\text{Stiffness } (k) = 2 \text{ N/mm} = 2 \frac{N}{mm\left(\frac{1}{1000}\right)}$$

$= 2 \times 1000$

$$= 2 \times 10^3 \text{ N/m}$$

$m \rightarrow kg$

$k \rightarrow N/m$

Angular frequency (ω)

$$\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{2 \times 1000}{1.529}}$$

$$\boxed{\omega = 36.17 \text{ rad/sec}}$$

Natmal frequency (f)

$f = \frac{1}{T}$

$$f = \frac{\omega}{2\pi} = \frac{36.17}{2\pi}$$

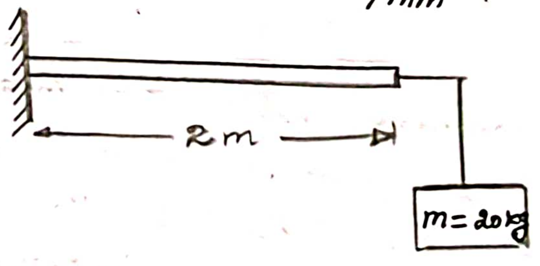
$f = 5.757$ Hertz

$$\boxed{f = 5.76 \text{ Hz}}$$

Formula

SL.NO	Beam	Stiffness
1.	Cantilever 	$k = \frac{3EI}{l^3}$
2	simply Supported 	$k = \frac{48EI}{l^3}$
3	Fixed beam 	$k = \frac{192EI}{l^3}$

② Find the natmal frequeny of the cantilever beam having a length of 2m and subjected to a mass of 20kg as shown in fig. The c/s area of the beam is 200 mm × 300 mm and $E = 2.2 \times 10^4$ N/mm².



Given Data :-

b = 200 mm

d = 300 mm

m = 20 kg

L = 2 m

$E = 2.2 \times 10^4$ N/mm²

Stiffness (k)

The deflection at the free end of the cantilever beam acted upon the static force at the free end.

$$\text{Deflection } (\Delta) = \frac{Wl^3}{3EI}$$

$$\text{Stiffness } (k) = \frac{W}{\Delta} = \frac{W}{\frac{Wl^3}{3EI}}$$

$$= \frac{3EI \cdot W}{Wl^3}$$

For cantilever beam $\boxed{k = \frac{3EI}{l^3}}$

$$I = \frac{bd^3}{12} = \frac{200 \times 300^3}{12}$$

$$\boxed{I = 450 \times 10^6 \ mm^4}$$

$$\text{Stiffness } (k) = \frac{3EI}{l^3}$$

$$= \frac{3 \times 2.2 \times 10^4 \times 450 \times 10^6}{(2000)^3}$$

$$\boxed{k = 3712.5 \ N/mm}$$

$$k = 3712.5 \times 10^3 \ N/m$$

Angular frequency (ω)

$$\omega = \sqrt{k/m} = \sqrt{\frac{3712.5 \times 10^3}{20}}$$

$$\boxed{\omega = 430.84 \ rad/sec}$$

Natural frequency (f)

$$f = \frac{\omega}{2\pi} = \frac{430.84}{2\pi}$$

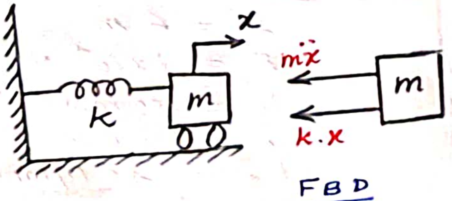
$$\boxed{f = 68.57 \ Hz}$$

---

③ A cantilever beam 3 m long supports a mass of 500 kg at its upper end. Find the natural period and natural frequency. Take $E = 2.1 \times 10^6 \ kg/cm^2$ & $I = 1300 \ cm^4$ & also Draw FBD (Free body diagram).

Given Data

$L = 3 \ m$
$m = 500 \ kg$
$E = 2.1 \times 10^6 \ kg/cm^2$
$I = 1300 \ cm^4$



FBD

Stiffness (k)

For cantilever beam, $k = \frac{3EI}{l^3}$

$$= \frac{3 \times 2.1 \times 10^6 \ \frac{kg}{cm^2} \times 1300 \ cm^4}{(300)^3 \ cm^3}$$

$1 N.m = 100 N.cm$
$1 N/cm = 100 N/m$

$$k = 303 \ kg/cm$$

$$= 303 \times 9.81 \ N/cm$$

$\frac{N}{cm} \times 100 = N/m$

$$k = 2.97 \times 10^3 \ N/cm$$

$$\boxed{k = 29.71 \times 10^4 \ N/m}$$

$k = 29.7 \times 10^4 \ N/m$

Natural frequency (f)

Angular frequency $\omega = \sqrt{\frac{k}{m}}$

$$= \sqrt{\frac{2.97 \times 10^5}{500}}$$

$$\boxed{\omega = 24.37 \ rad/sec}$$

Natural frequency $(f) = \frac{\omega}{2\pi}$

$$= \frac{24.37}{2\pi}$$

$$\boxed{f = 3.88 \ cps}$$

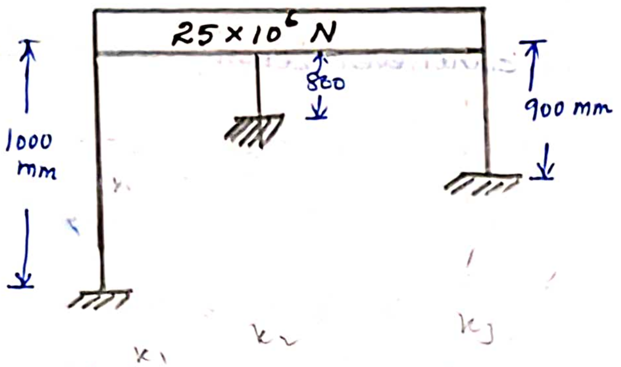
Natural period (T)

$$T = \frac{1}{f} = \frac{2\pi}{\omega} = \frac{2\pi}{24.37}$$

$$\boxed{T = 0.26 \ sec}$$

---

④ Find the nature of frequency and natural period of vibration of the frame as shown in fig. The initial displacement is 25 m & initial velocity is 25 mm/sec. Also find displacement at time $t = 1 \ sec$; $EI = 30 \times 10^{12} \ N.mm^2$



Given Data:-

Initial displacement $x_0 = 25\,mm$

Initial velocity $\dot{x}_0 = 25\,mm/s$

$L_1 = 1000mm$

$L_2 = 800mm$

$L_3 = 900\,mm$

$EI = 30 \times 10^{12}\ N \cdot mm^2$

Mass $(m) = \frac{W}{g} = \frac{25 \times 10^6}{9.81}$

$m = 2.54842 \times 10^6\ kg$

Stiffness (k)

$$k_1 = \frac{12EI}{l_1^3} = \frac{12 \times 30 \times 10^{12}}{1000^3} = 360 \times 10^3\ N/mm$$

$$k_2 = \frac{12EI}{l_2^3} = \frac{12 \times 30 \times 10^{12}}{800^3} = 703.13 \times 10^3\ N/mm$$

$$k_3 = \frac{12EI}{l_3^3} = \frac{12 \times 30 \times 10^{12}}{900^3} = 493.83 \times 10^3\ N/mm$$

Since, the stiffness are parallel

$\therefore\ k_e = k_1 + k_2 + k_3$

1 N/mm = 1000 N/m

$k_e = 360 \times 10^3 + 703.13 \times 10^3 + 493.83 \times 10^3$

$\boxed{k_e = 1.55 \times 10^6\ N/mm} = 1.55 \times 10^9\ N/m$

Angular frequency (ω)

$$\omega = \sqrt{k/m} = \sqrt{\frac{1.55 \times 10^9}{2.548 \times 10^6}}$$

$\boxed{\omega = 24.66\ rad/sec}$

$\boxed{\omega = 24.66\ rad/sec}$

Natural frequency (f)

$$f = \frac{\omega}{2\pi} = \frac{24.66}{2\pi}$$

$\boxed{f = 3.925\ CPS\ or\ Hz}$

Natural Period of vibration (T)

$$T = \frac{1}{f} = \frac{1}{3.925}$$

$\boxed{T = 0.26\ sec}$

Determination of displacement (x)

$$x = A\cos\omega t + B\sin\omega t$$

$$x = x_0\cos\omega t + \frac{\dot{x}_0}{\omega}\sin\omega t$$

$$= 25\cos(24.66 \times 1) + \frac{25}{24.66}\sin 24.66 \times 1$$

$$x = 22.7200 + 0.42300$$

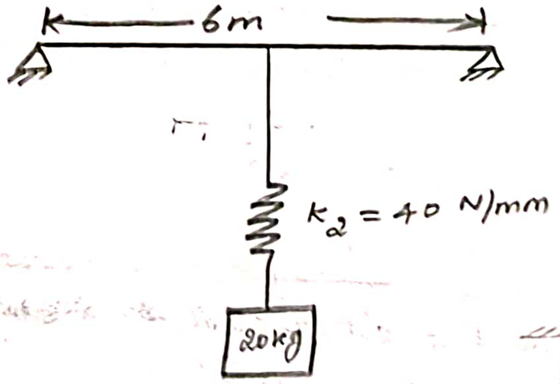
$\boxed{x = 23.1434\ mm}$

---

⑤ Find the natural frequency of the system as shown in fig.

$E = 2 \times 10^4\ N/mm^2$.

Size of the beam is $100 \times 150\ mm$



Given Data

$L = 6\ m$

$E = 2 \times 10^4\ N/mm^2$

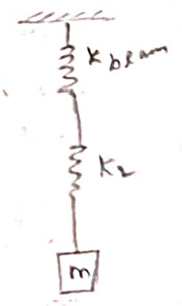
$b = 100\ mm$

$d = 150\ mm$

$k_2 = 40\ N/mm$

$m = 20\ kg = 20 \times 9.81$

$m = 196.2\ N$



$\overline{K/m}$

K/m

stiffness (k)

For ss beam,

$$k_1 = \frac{48EI}{l^3}$$

$$I = bd^3/12 = \frac{100 \times 150^3}{12}$$

$$I = 28.125 \times 10^6 \ mm^4$$

$$= \frac{48 \times 2 \times 10^4 \times 28.125 \times 10^6}{(6000)^3}$$

$$\boxed{k_1 = 125 \ N/mm}$$

Given, $k_2 = 40 \ N/mm$

since the springs are in series

$$\therefore \frac{1}{k_e} = \frac{1}{k_1} + \frac{1}{k_2}$$

$$\frac{1}{k_e} = \frac{1}{125} + \frac{1}{40} = 0.0333\ 0.025$$

$$\frac{1}{k_e} = \frac{1}{0.033}$$

$$\therefore \boxed{k_e = 30.30 \ N/mm}$$

Angular frequency (ω)

$$\omega = \sqrt{K/m} = \sqrt{\frac{30.30}{20 \times 9.81}}$$

$$\boxed{\omega = 0.392 \ rad/sec}$$

Natural frequency (f)

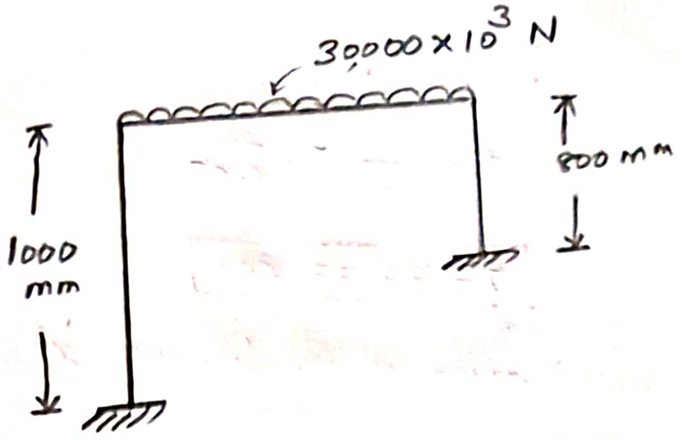
$$f = \frac{\omega}{2\pi} = \frac{0.392}{2 \times \pi}$$

$$\boxed{f = 0.06 \ Hertz}$$

---

⑥ calculating the nature of frequency for the frame as shown in fig. and also nature of period of vibration. If the initial displacement is 25 mm & initial velocity 25 mm/sec What is the amplitude and displacement @ $t = 1$ sec, $EI = 30 \times 10^{12}$ N.mm^2

(c)



Given Data :-

initial displacement $x_0 = 25$ mm ; $L_1 = 1000$ mm

initial velocity $\dot{x}_0 = 25$ mm/sec ; $L_2 = 800$ mm

$EI = 30 \times 10^{12}$ N/mm^2

$t = 1$ sec

Mass $(m) = 30{,}000 \times 10^3$ N

Stiffness (k)

For springs in parallel,

$$\therefore k_e = k_1 + k_2$$

$$k_1 = \frac{12EI}{l_1^3} = \frac{12 \times 30 \times 10^{12}}{(1000)^3} = 36 \times 10^4 \ N/mm$$

$$k_2 = \frac{12EI}{l_2^3} = \frac{12 \times 30 \times 10^{12}}{(800)^3} = 703.13 \times 10^3 \ N/mm$$

$$k_e = 360 \times 10^3 + 703.13 \times 10^3$$

$$\boxed{k_e = 1063.13 \times 10^3 \ N/mm}$$

Angular frequency (ω)

$$\omega = \sqrt{K/m} = \sqrt{\frac{1063.13 \times 10^3}{30{,}000 \times 10^3}}$$

$$\boxed{\omega = 0.19 \ rad/sec}$$

Natural frequency (f)

$$f = \frac{\omega}{2\pi} = \frac{0.19}{2\pi}$$

$$\boxed{f = 0.030 \ hertz}$$

Time period (T)

$$T = \frac{1}{f} = \frac{1}{0.03}$$

$$\boxed{T = 33.33 \text{ sec}}$$

Determination of Displacement (x)

$$x = A\cos\omega t + B\sin\omega t$$

$$x = x_0\cos\omega t + \frac{\dot{x}_0}{\omega}\sin\omega t$$

$$= 25 \times \cos 0.19 \times 1 + \frac{25}{0.19}\sin 0.19 \times 1)$$

$$\boxed{x = 25.43 \text{ mm}}$$

Amplitude (A)

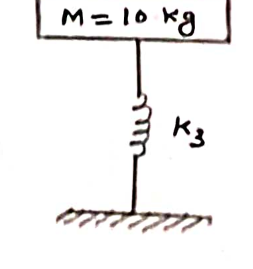
$$A = \sqrt{x_0^2 + \left(\frac{\dot{x}_0}{\omega}\right)^2}$$

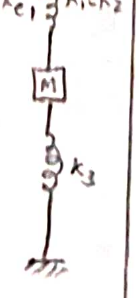
$$= \sqrt{(25)^2 + \left(\frac{25}{0.19}\right)^2}$$

$$\boxed{A = 135.3 \text{ mm}}$$

---

⑦ Find the natural frequency of the system as shown in fig. Take $k_1 = k_2 = 2000$ N/m ; $k_3 = 3000$ N/m & $m = 10$ kg.





Given Data:-

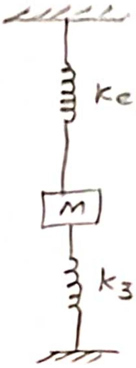
$$k_1 = k_2 = 2000 \text{ N/m}$$
$$k_3 = 3000 \text{ N/m}$$
$$m = 10 \text{ kg.}$$

Stiffness (k)

Two sprinks k_1 & k_2 are in Paralle

$$\therefore k_{e_1} = k_1 + k_2$$
$$= 2000 + 2000$$
$$\boxed{k_{e_1} = 4000 \text{ N/m}}$$

Again this equivalent Spring Is Parallel to k_3



$$\therefore k_e = k_{e_1} + k_3$$
$$= 4000 + 3000$$
$$\boxed{k_e = 7000 \text{ N/m}}$$

Natural frequency (f)

$$\omega = \sqrt{\frac{K}{m}} = \sqrt{\frac{7000}{10}}$$

Angular frequency $\boxed{\omega = 26.46 \text{ rad/sec}}$

$$\text{Natural frequency } (f) = \frac{\omega}{2\pi}$$
$$= \frac{26.46}{2\pi}$$
$$\boxed{f = 4.21 \text{ Hz}}$$

---

⑧ Determine the equivalent spring stiffness and natural frequency of vibrating system as shown in fig.

(a) The mass is suspended to a spring

(b) The mass is suspended at the bottom of two springs in series

(c) The mass is fixed in b/w two springs.

(d) The mass is fixed to the mid point of a spring

(e) The mass is suspended at the bottom of two springs in parallel.

Take, $k_1 = 1500$ N/m

$k_2 = 900$ N/m

$m = 12$ kg.

Solution:-

(a) Mass is suspended to a spring

$k_e = k_1 = 1500$ N/m

$m = 12$ kg.

Angular frequency $(\omega) = \sqrt{\frac{K}{m}} = \sqrt{\frac{1500}{12}}$

$\boxed{\omega = 11.18 \text{ rad/sec}}$

Natural frequency $(f) = \frac{\omega}{2\pi} = \frac{11.18}{2\pi}$

$\boxed{f = 1.78 \ Hz}$

(b) The mass is suspended at the bottom of two springs in series

Springs in series,

$\therefore$ Stiffness, $k_e = \frac{k_1 k_2}{k_1 + k_2}$

$k_e = \frac{1500 \times 900}{1500 + 900}$

$\boxed{k_e = 562.5 \text{ N/m}}$

Angular frequency $(\omega) = \sqrt{K/m} = \sqrt{\frac{562.5}{12}}$

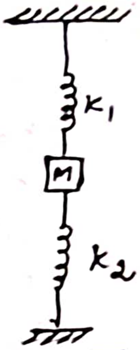
$\boxed{\omega = 6.85 \text{ rad/sec}}$

Natural frequency $(f) = \frac{\omega}{2\pi} = \frac{6.85}{2\pi}$

$\boxed{f = 1.09 \ Hz}$

(c) The mass is fixed in b/w two springs

Net spring force = spring force in spring 1 + spring force in spring 2



$kx = k_1 x_1 + k_2 x_2$

$\therefore k_e = k_1 + k_2$

$= 1500 + 900$

$\boxed{k_e = 2400 \text{ N/m}}$

Angular frequency $(\omega) = \sqrt{K/m} = \sqrt{2400/12}$

$\boxed{\omega = 14.14 \text{ rad/sec}}$

Natural frequency $(f) = \frac{\omega}{2\pi} = \frac{14.14}{2\pi}$

$\boxed{f = 2.25 \text{ Hertz}}$

(d) The mass is fixed to the mid point of a spring.

Stiffness of spring on each side $= 2k$

(or)

$k_e = 2k_1 + 2k_2$

$= 4k_1$

$k_e = 4 \times 1500$

$\boxed{k_e = 6000 \text{ N/m}}$

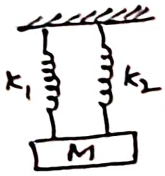
Angular frequency $(\omega) = \sqrt{\frac{k}{m}} = \sqrt{\frac{6000}{12}}$

$\boxed{\omega = 22.361 \text{ rad/sec}}$

Natural frequency $(f) = \frac{\omega}{2\pi} = \frac{22.361}{2\pi}$

$\boxed{f = 3.56 \text{ Hertz}}$

(e) Mass is suspended at the bottom of two springs is parallel.



$k_e = k_1 + k_2$

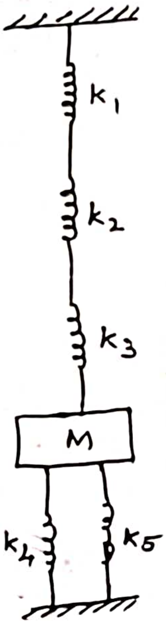
$= 1500 + 900$

$\boxed{k_e = 2400 \text{ N/m}}$

$\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{2400}{12}} = 14.14$ rad/sec

$f = \frac{\omega}{2\pi} = \frac{14.14}{2\pi} = 2.25$ Hertz

⑧ For the system as shown in fig. $k_1 = k_2 = 500$ N/m ; $k_3 = 1500$ N/m ; $k_4 = 3000$ N/m $k_5 = 2000$ N/m. Find the mass 'm' such that the system has a natural frequency of 6.75 Hertz.



Solution

Stiffness (k)

Springs in series

$$\frac{1}{k_{e_1}} = \frac{1}{k_1} + \frac{1}{k_2} + \frac{1}{k_3}$$

$$= \frac{1}{500} + \frac{1}{500} + \frac{1}{1500}$$

$$\frac{1}{k_{e_1}} = 0.004667$$

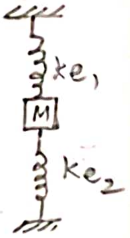
$$\therefore \boxed{k_{e_1} = 214.286 \text{ N/m}}$$

Springs in parallel

$$k_{e_2} = k_4 + k_5$$

$$= 3000 + 2000$$

$$\boxed{k_{e_2} = 5000 \text{ N/m}}$$



$\therefore$ Total stiffness $k_e = k_{e_1} + k_{e_2}$

$$\boxed{k_e = 5214.286 \text{ N/m}}$$

Angular frequency (ω)

$$\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{5214.286}{m}}$$

Natural frequency (f)

$$f = \frac{\omega}{2\pi} = \frac{\sqrt{\frac{5214.286}{m}}}{2\pi}$$

$$6.75 = \frac{72.21 \times \left(\frac{1}{m}\right)^{1/2}}{2\pi}$$

$$\frac{42.411}{72.21} = \left(\frac{1}{m}\right)^{1/2}$$

$$0.5873 = \left(\frac{1}{m}\right)^{1/2}$$

$$(0.5873)^2 = \frac{1}{m}$$

$$0.345 = 1/m$$

$$\therefore m = \frac{1}{0.345}$$

$$\boxed{m = 2.899 \text{ kg}}$$

⑨ A mass of 1 kg is suspended by a spring having a stiffness of 600 N/m. The mass is displaced downward from its equilibrium position by a distance of 0.01 m. Find

(a) Equation of motion of the system
(b) Natural frequency of the system
(c) The response of the system as a function of time
(d) Total energy of the system.

Given Data:-

$m = 1$ kg
$k = 600$ N/m
$\delta_{st} = 0.01$ m.

(a) Equation of motion is given by

$$m\ddot{x} + kx = 0$$

$$\boxed{1\ddot{x} + 600x = 0}$$

(b) The natural frequency of the system

$$\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{600}{1}}$$

$$\boxed{\omega = 24.49 \text{ rad/sec}}$$

$$f = \frac{\omega}{2\pi} = \frac{24.49}{2\pi}$$

$$\boxed{f = 3.898 \text{ Hz}}$$

(c) Response of the system as a function of time

$$x = A\sin(\omega_n t + \xi)$$

Amplitude $A = \sqrt{x_0^2 + \left(\frac{\dot{x}_0}{\omega}\right)^2}$

$$= \sqrt{(0.01)^2}$$

$$\boxed{A = 0.01 \text{ m}}$$

Phase angle $(\xi) = \tan^{-1}\left(\frac{x_0\omega}{\dot{x}_0}\right)$

$$= \tan^{-1}\left(\frac{0.01 \times 24.49}{0}\right) = \tan^{-1}(\infty)$$

$$= \pi/2$$

Response $x = 0.01\sin\left(24.49t + \frac{\pi}{2}\right)$

(d) Total energy

Total energy = Max. kinetic energy (or) Max. Potential energy

$$PE_{max} = \frac{1}{2}k.\cancel{x}^2 = \frac{1}{2} \times 600 \times 0.01^2 = 0.03 \text{ N/m}$$

$$KE_{max} = \frac{1}{2}mv^2 = \frac{1}{2} \times m.(A\omega)^2 = \frac{1}{2} \times 1 \times (24.49 \times 0.01)^2$$

$= 0.07$ N/m [illegible] $= 0.03$ N/m

Unit - II - Elements of Seismology.

Elements of seismology — causes of Earthquake — plate tectonic theory — Elastic rebound theory — characteristic of Earthquake — estimation of earthquake parameters — magnitude and intensity of earthquakes — spectral acceleration.

Seismology:-

* It is the science [links physics with other geosciences (geology, geography)] dealing with all aspects of Earthquakes

* Types of seismology

1) observational seismology
2) Engineering seismology
3) physical seismology.

observational seismology:-

* Recording earthquake (micro seismology)

* cataloguing earthquake

* observing earthquake effects (macro seismology)

Engineering seismology:-

* Estimation of seismic hazard and risk.

* Study & Design of Aseismic building

physical ~~Geology~~ seismology:-

* study of the properties of the Earth's interior

* Study of physical characteristics of seismic sources.

Earthquake

* Earthquake is a sudden tremor or movement of the earth's crust, which originates naturally at or below the surface.

Seismology:-

* It is the study of ~~the~~ generation, propgation and recording of elastic waves in the earth and the sources that produce them.

Classification of Earthquake

Earthquake

- Based on Location (or) plate boundaries
  - * Interplate EQ
  - * Inttraplate EQ
- Based on focul depth
  - * shallow focus EQ
  - * Intermediate EQ
  - * Deep focus EQ
- Based on origin of Earthquake
  - * Tectonic EQ
  - * Volcanic EQ
  - * Submarine EQ
- Based on Magnitude
  - * Micro EQ
  - * Intermedia
  - * Moderate
  - * strong
  - * Major
  - * Great

a) Based on Location or plate boundaries

1) Inter plate earthquake → EQ occuring along the boundaries of the tectonic plate.
Example :- 1897, Assam earthquake

2). Intra plate earthquake → EQ occuring with in a plate
Example : 1993, Latur earthquake.

b) Based on depth of focus

1) shallow focus earthquake → seismic shocks originates at a depth of about less than 70km
* 80% world earthquakes - shallow EQ.

2) Intermediate focus earthquake → seismic waves originates at a depth b/w 70 km to 300km

3) Deep focus earthquake → seismic waves originates at a depth of greater than 300km.

c) Based on origin of earthquake :-

1) Tectonic earthquakes → It is an earthquake indued by the movement (injection or withdrawal) of magma.

Pressure of plat movements exceeding the pressure

* The movement results in pressure changes in the rock around where the magma has experienced stress.
* At the point, the rock may break or move.

⊛ * occurs, when rocks in the earth crust break due to the geological forces created by movement of tectonic plates.

2) Volcanic Earthquake → occurs in conjunction with volcanic activity.

Effects of Earthquake

- * Damage to building → (Glass Items falls. windows, mirrors etc) Complete collapse of building, (dead to humans)
- * Damage to infrastructure → EB line, road, pipeline, gas lin result fire & explosion.
- * Land slides & rock slides → large rocks, uphill, rolling rapidly down in to the valley.
- * Floods → cracking of dam wall - Land slide → death to people near by areas
- * Trigger Tsunamis — volcanic eruptions under the sea
- * Liquefaction of soil → soil saturated & loses it strength

Factors influencing Ground motion

- * Magnitude of earthquake → higher the magnitude, large is the peak ground acceleration & duration.
- * Epicentral distance → PGA decreases, epicentral distance increases.
- * Local soil condition → soil layers overlaying the bed at a given place, change the characteristics of the waves in terms of amplitude, frequency and duration by the time.

Estimation of Earthquake parameters

- * Ground shaking is recorded with an instrument called seismometer.
- * The instruments make a recording on a device

3) Submarine Earthquake → occurs under water at the bottom of a body of water, especially an ocean.

* They are leading cause of tsunamis.

d) Based on magnitude

1) Micro Earthquake → $M < 3$
2) Intermediate Earthquake → $3 < M < 4$
3) Moderate earthquake → $5 < M < 5.9$
4) Strong earthquake → $6 < M < 6.9$
5) Major earthquake → $7 < M < 7.9$
6) Great earthquake → $M > 8$

Characteristic of Earthquake Ground motion

* Affect human being and their environment — strong ground motion.

* Strong ground motion measured by accelerographs and its records time history of accelerogram.

* The ground motion represented in terms of displacement, velocity and acceleration.

* Parts of accelerogram —
- Rise
- strong motion
- decay.

* The fault is strongly, & it is dependent on nature.

* Earthquake motion depends upon fault shape, area or location, maximum fault dislocation and Stress drop fault plane.

* Amplitude properties → Horizontal component acceleration

* Duration

* Effect of distance

* Ground motion level → ~~geo~~ geological, Geophysical and Geotechnical data.

Earthquake

Based on Location

1. Inter plate EQ → occurs tectonic plate boundary
2. Intraplate EQ → occurs within plate

Based on focal depth

1. shallow focus EQ → Waves originate Less than 70km
2. Intermediate → 70 to 300km
3. Deep → Waves originate greater than 300km

Based on origin of EQ

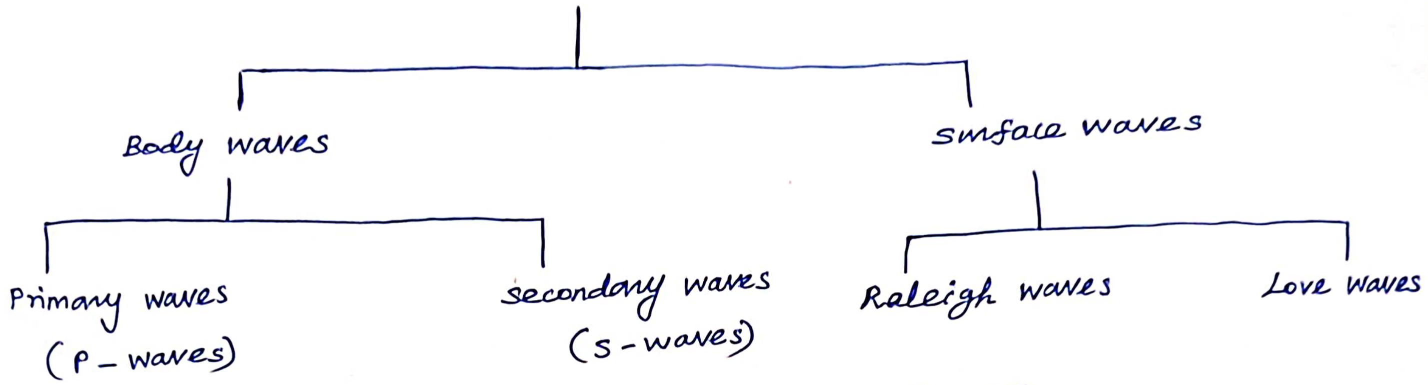
1. Tectonic EQ → Pressure & tectonic plate movement
2. Volcanic EQ → occurs volcanic activity
3. submarine EQ → occurs under water

Based on Magnitude

Based on magnitude

1. Micro EQ → magnitude less than 3
2. Intermediate → magnitude 3 to 3.9
3. Moderate → 5 to 5.9
4. Strong → 6 to 6.9
5. Major → 7 to 7.9
6. Great → greater than 8

Seismic waves



Primary waves (P – waves)

- * Travel faster than S-waves
- * Longitudinal (or) compressional
- * Travel through liquid (or) solid
- * Double times speed of S-waves
- * In water velocity – 1450 m/s
- * In granite – 5000 m/s

Secondary waves (S – waves)

- * Travel slower than P-waves
- * Transverse (or) shear wave
- * Travel only solids
- * can not travel liquid
- * 60% speed of P-waves.
- * Granite – 3.0 km/sec

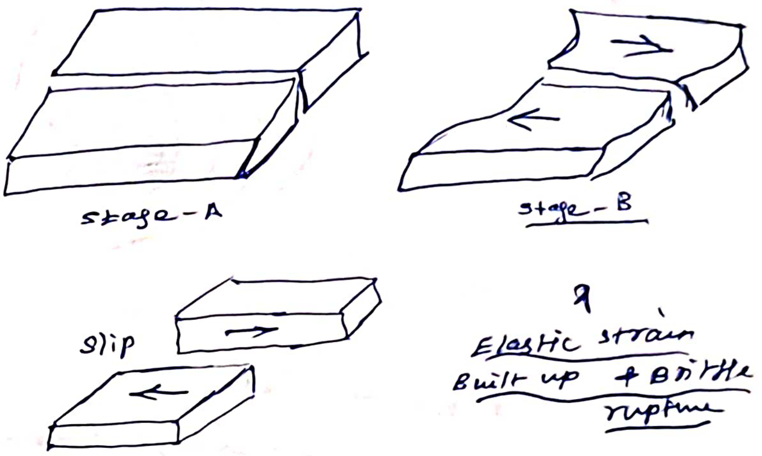
Rayleigh waves

- * oscillate in elliptic path (vertical plane)
- * Particle motion — vertical & horizontal
- * Particle below the free surface upto a depth equal to wave length.

Love waves

- * Maximum damage to structure
- * Particle motion — horizontal plane & transverse direction
- * oscilliate sidways in horizontal plane
- * Bottom of soil layer generate horizontally travelling.

1) Elastic Rebound theory:-



* Rocks are made of elastic material.

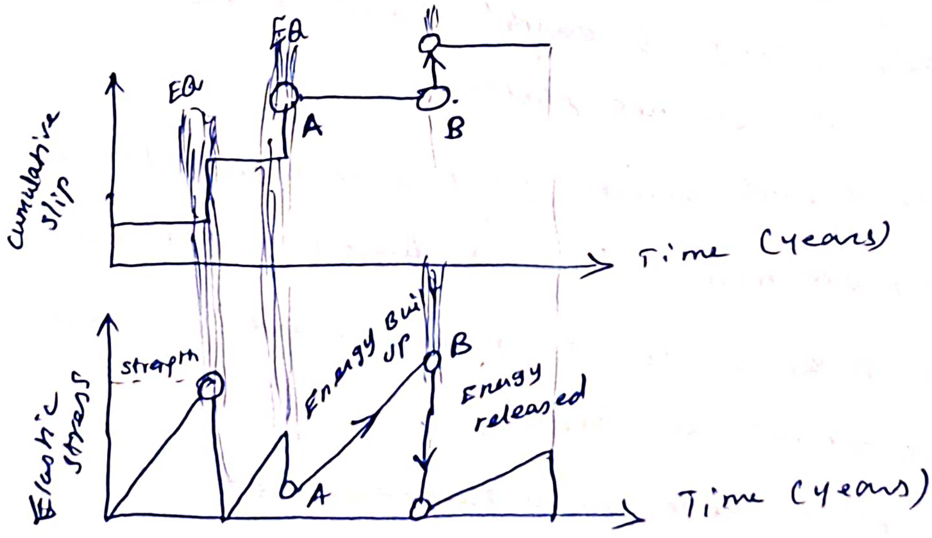
* Elastic energy is stored

* then elastic deformation occurs in very large (earth crust) tectonic plate actions occurs in earth.

* Rocks are brittle

* rock — weak region — earth crust reach their strength sudden movement — then fault (rocks are cracked) slip are formed caused earthquake.

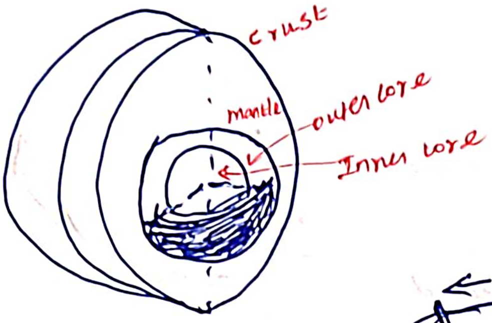
* Example 2001 - Bhuj earthquake is more strain energy released. (compared to ~~Bhuj~~ is 400 time large on Bhuj 1945 Atom bomb dropped on Hiroshima)

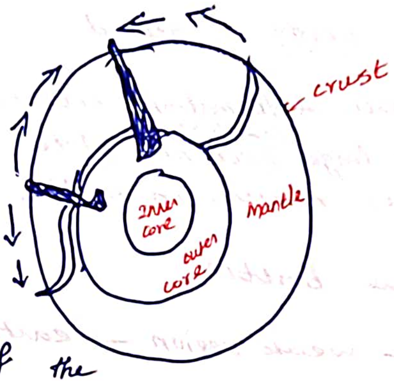


2) Tectonic plate theory:

* (The transferent of heat with in a fluid) The convective flows of mantle material cause the crust.

*





* Due to heat
* the mantle → high temperature
  & high pressure
* molten lava comes out & the cold rock mass goes in to the earth

* The convective flows of mantle material cause the crust & some portion of mantle to slide on the hot molten outer core.
  This sliding of earth's mase takes place in pieces are called Tectonic plates.

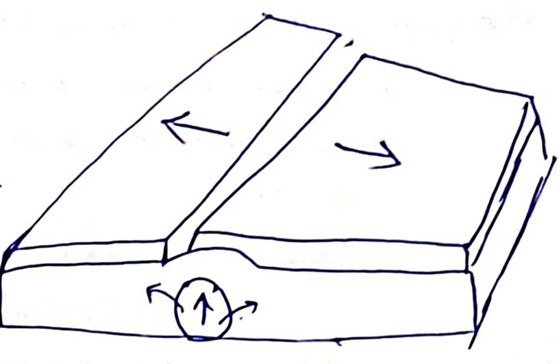
* surface of earth consists of 7 major tectonic plates & many smaller one.
* These plates move in different directions & different speeds.
* Plate is the slower, the plate behind it comes & collides mountains are formed.
* two plates move away from one another → rift (valley) formed
* Two plates move side by side, along same direction or opposite direction

↓ convergent → divergent → transform boundaries.

convergent ↓ Himalayas

3) occurence of Earthquakes

a) Divergent boundaries



* plates are moving apart a new crust is created by upward movement of molten magma.

* These distribution of earthquake – narrow band – activity on oceanic ridge (or) rift zone.

* Earthquake occurs in shallow depth (2 to 8 km).

* Magnitude greater than six is rare

* lithosphere is very thin & weak at divergent boundaries, so the strain energy built up is not enough to cause ~~an~~ large earthquake.

b) convergent boundaries

* Earth's unchanging size implies the crust destroyed ~~about~~ ~~the~~ at Convergent boundaries sea floor spreading hypothesis

Tsunami

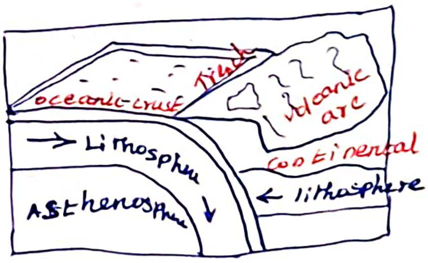
* Convergent boundaries Plate moving toward each other & one plate sinks under another.

* sinking of a plate occurs in called subduction zone.

* convergence occur b/w an oceanic & continental plates or b/w two oceanic plate

* ten largest earthquakes since 1900 on the globe along subdution zones & including 26th Dec 2004 earthquake in Indonesia of massive tsunami,

c) oceanic-continental convergence



* It is long narrow, curving trench thousands of kilometers long & to 10 km deep cutting in ocean floor

* ~~Higher density~~ At this ocean Higher density

* so strong, destructive earthquakes and rapid uplift mountain ranges.

*

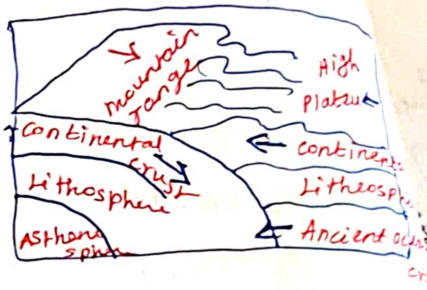
d) **Oceanic - oceanic convergence**

- * two oceanic plates converge, older one is usually subdeducted under the other & in the ~~this~~ process a trench is formed.

<u>Example</u> Mariana's treoh in fast moving

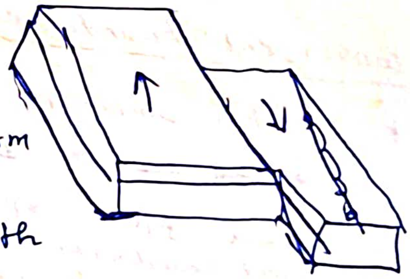
philippine plate - slow moving

e) **continental - continental convergence**



- * Himalayan mountain
- * It relatively light & like two colliding icebergs, resist downward motion.
- * The crust tends to buckle and be pushed upward or side ways.
- * About 40 to 50 million years ago the boundary b/w Indian Plate and the Eurasian plate was oceanic - continental nature

f) **Transform boundaries**



- * zone b/w two plates sliding horizontally past one another is called a transform fault boundary.
- * earthquake occur at shallow depth
- * friction b/w the plates can be so great that very large strains.

---

4) Seismic waves

Body waves, Surface waves

Body waves: Primary waves P(waves) Compression; Secondary waves (S-waves) Shear

- * Similar to sound waves
- * obey physical laws
- * Mass particle motion of P-wave in the direction of propogation of the wave
- * It is fastest wave

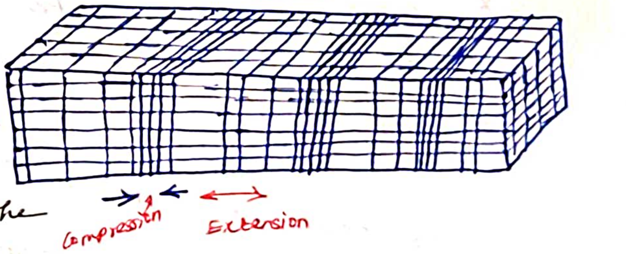
- * velocity of S-wave directly related to shear strength
- * move in a direction ⊥r to the direction of particle motion
- * waves are travels moves slowly.
- * s waves do not propogate through the fluid.

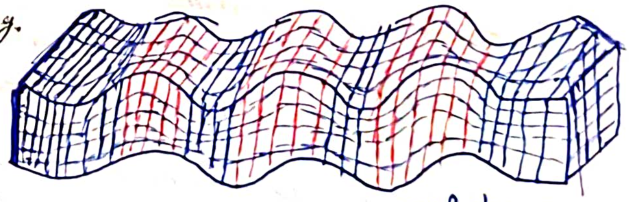
~~Defn~~

<u>S-Waves</u>

A seismic body wave that shakes the ground back & forth ⊥r to the direction of wave moving.

<u>P-waves</u> - Push & Pull





<u>S-waves</u> up & down side to side

<u>Defn</u>

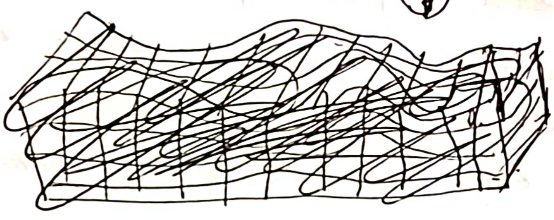
<u>P-wave</u> :- A seismic body wave that shakes the ground back and forth in the same direction and the opposite direction as the wave is moving.

Surface waves

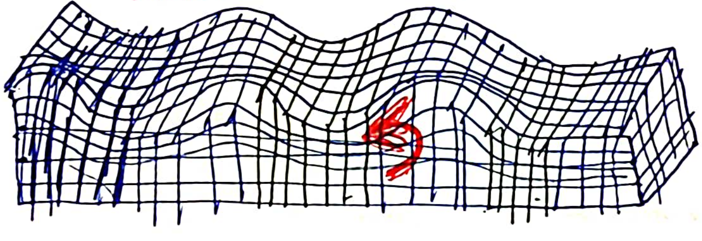
(a) Rayleigh waves

- * Vertical + horizontal components of particle motion are 90°
- * wave propogate → minor axis + major vertical axis. (ellipse) in vertical plane
- * The particle below the free surface up to a depth equal to wavelength
- * The amplitude of particle displacement decreases with depth.

particle motion



Elliptic in Vertical Plane

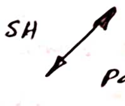


$V_{L_R} = 0.92\beta$

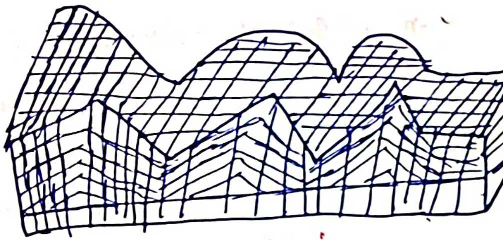
Defn A seismic surface waves causing the ground to shake is
elliptical motion with no transverse (or) ⊥r motion.

(b) Love waves

- * propogation of Love waves in horizontal soil layer overlaying the half-space
- * particle motion → horizontal plane + transverse to the direction
- * Bottom of the soil layer generate horizontally travelling Love waves.



particle motion



side ways in horizontal plane

Defn
A type of seismic surface waves having a horizontal motion i.e., transverse (or) ⊥r to the direction of the wave is travelling

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5) Classification of Earthquake

1 Based on location
- a) Interplate
- b) Interaplate

2) Based on focal depth
- a) shallow depth (0 to 71 km)
- b) Intermediate depth (71-300 km)
- c) Deep earthquake (> 300 km)

3) Based on magnitude
- a) Micro earthquake < 3
- b) Intermediate Earthquake 3 to 4
- c) moderate earthquake 5 to 5.9
- d) strong earthquake 6 to 6.9
- e) Major earthquake 7 to 7.9
- (f) Great earthquake > 8

4) Based on Epicentral distance
- a) Local earthquake < 1
- b) Regional Earthquake - 1 to 10
- c) Teleseismic earthquake > 10

6) Seismic hazard Map

West 1937

- * danger zone → All past earthquakes causing severe damage (MM intensity X and above) since 1850.
- * moderate zone damage → caused by earthquakes originating danger zone Damages severe damage close to epicentre region.
- * areas of comparative safe zone of slight or no damage.

Zone-1 → Frequient EQ zone
2 → occasional "
3 → Few (or) rare EQ. zone

1904 to 1950 Jai krishna (1958 & 1959)

- * very heavy damage zone → magnitude 8
- * heavy damage zone → max. acceleration 0.3g due to an epicentre of magnitude earthquake along southern margin
- * moderate damage zone → ground acceleration b/w ~~less than~~ 0.1g – 0.3g
- * Mithal & Srivastava (1959) classified occurrence of earthquakes in India on thickness of continental shelf using geophysical data based on
  - Assam (1897 + 1950) ~~&~~ Bihar - Nepal (1934) earthquakes.
  - Kangra (1905)

thickness more than 1500.

———— × ———— × ———— × ————

7) Characteristic of Earthquake Ground motion

- * affect human & their environment — strong ground motion
- * strong motions measured by accelerographs & its record is time history of accelerogram.
- * Accelerogram → 3 parts (i) rise, (ii) strong motion (iii) decay.
- * The fault & is strongly dependent on the nature
- * motion depends → fault shape, its area, max. fault dislocation & stress drop fault plane

1) Amplitude properties * Horizontal component acceleration	2) Duration	3) Effect of distance
4) Ground motion level	Geological, Geophysical & Geotechnical Data	

✓

Spectral Acceleration :-

- Strong ground motion from earthquakes are measured using PGA (Peak Ground Acceleration), PGV (Peak Ground velocity), Pseudo spectral acceleration or Velocity and intensity.

- severity of ground shaking increases with magnitude and decreases with epicentral distances.

- It is enhanced in the directions of rupture propagations.

- Low velocity soil site gives much higher ground motion than rock site.

- spectral acceleration, with a value related to the natural frequency of vibration of the building, is used in earthquake engineering.

- The release of the accumulated elastic strain energy by the sudden rupture of the fault is cause of earthquake shaking.

- Ground motions are caused by seismic waves generated by the release of strain energy

- The waves are travel with different velocity amplitudes and levels of energy.

Peak Ground Acceleration (PGA).

- It is the maximum acceleration which is experienced by particle on the ground.

- Example :- Ground acceleration recorded in Northridge is 0.8g, which represents the movement of the ground.

- PGA is easy to measure because the response of most instruments is proportional to ground acceleration.

* It is a convenient single number to enable rough evaluation of importance of records.

Peak Ground Velocity (PGV)

* It is sensitive to longer periods than PGA but it requires digital processing.

Peak Ground Displacement (PGD)

* It is the best parameter for displacement based design but highly sensitive to the low cut filter that needs to be applied to most records.

In case of earthquake resistant design of structures

* the ground acceleration is the most significant parameter of strong motion being directly proportional to the inertia force imposed on the structures.

* In IS code of practice, the vertical acceleration is taken as $\frac{1}{2}$ to $\frac{2}{3}$ of the horizontal design acceleration.

Unit - III.

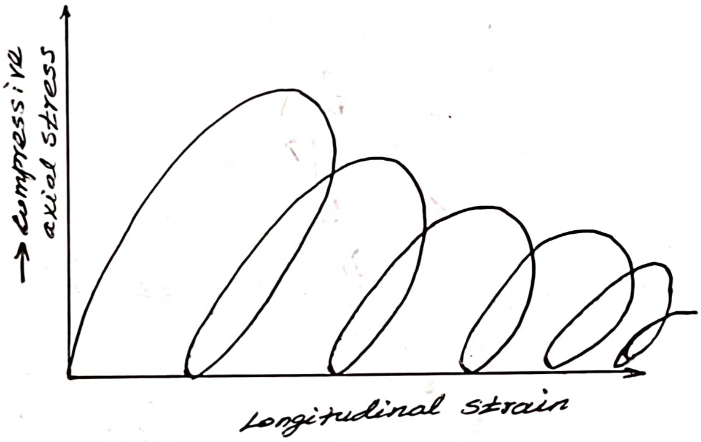
Cyclic behaviour of concrete and Reinforcement with respect to pinching & Bouchinger effect.

Plain concrete

* It is a brittle material

* During the first cycle — the stress strain curve is the same as that obtained from static tests.

* If the specimen is unloaded & reloaded in compression, stress strain curve similar to fig.



* It can be seen that slope of the stress strain curves as well as maximum attainable stress decreases with number of cycles.

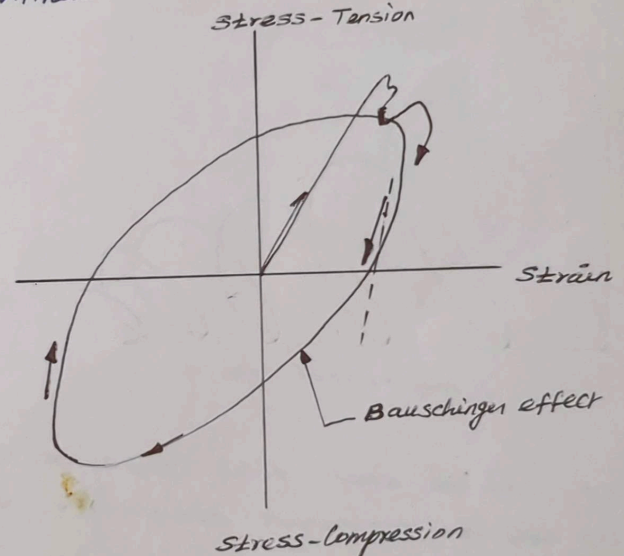
* Thus the stress strain relationship of plain concrete subjected to repeated compressive loads is cycle dependent.

* The decrease in stiffness and strength of plain concrete is due to formation of cracks.

* plain concrete cannot subjected to repeated tensile loads since its tensile strength is practically zero.

Reinforcing steel:-

- * Reinforcing steel has much more ductility than plain Concrete.
- * Ultimate Strain in mild steel is of order of 25% where as in Concrete it is the order of 0.3%.
- * In the first cycle, the reinforcing Steel shows Stress strain curve Similar to that obtained in the Static test.
- * After the specimen has reached its yield level and direction of load is reversed, that is unloading begins, it is show in fig. that the unloading curve is not straight but curvilinear.



- * This curvature in the unloading segment of stress strain curve is referred as Bouchinger effect.
- * The figure shows the one complete cycle of loading and unloading referred as hysteresis loop.
- * The area with in the hysteresis loop exhibits energy absorbed by the specimen in a cycle. In subsequent cycles the same path is repeated.
- * Thus the stress strain relationship of Mild reinforcing steel subjected to repeated reverse loading is cyclic independent until the specimen buckles or fails due to fatigue.

- Response of the Structure to ~~Response~~ Earthquake

- * Response spectrum is the fundamental of earthquake response analysis for structures.
- * The structural response is vibratory (dynamic) and it is cyclic about equilibrium position of structure.
- * For most civil Engg. structures, fundamental natural frequency lies in the range of 0.1 sec to 3 sec.
- * It is the range of earthquake generated ground motion.
- * To perform the seismic analysis and design of structures to be built at a particular location, the actual time history record is required.
- * It is not possible to have such records at each & every location.
- * Seismic analysis of structures depend upon the frequency content of ground motion & its own dynamic properties to overcome such difficulties, earthquake response spectrum is the most popular tool in the seismic analysis of structure.
- * Computerized advantages in using the response spectrum method, in structural system
  - * Prediction of displacements
  - * Prediction of member forces
- * Method involves calculation of only maximum values of displacement & members forces in each mode of vibration.

Effect of earthquake damage depends upon

- Initensity
- Duration
- frequency of ground motion
- Geologic & soil condition
- quality of construction.

Behaviom of Reinforced cement concrete, steel and Prestressed concrete structures under earthquake loading

1) Mud and adobe Houses

- Unburnt sun dried bricks laid in mud morton are called adobe construction.
- Mud houses are the traditional construction for poor, and most suitable in view of their initial cost, easy availability, Low level skill construction and exellent insulation against heat and cold.
- More than 100 million people in India live in these type of houses.
- Examples of complete collapse of such building, 1906 Assam, 1948 Ashkhabad, 1960 Agadir, 1966 Tashkent, 1967 Koyna, 1975 kinnaur, 1979 Indo-Nepal, 1980 Jammu & kashmir, 1982 Dhamar earthquakes
- It is very weak in shear, torsion and compression.
- seperation of walls at corner and junctions takes place easily under ground shaking.
- The cracks pass through the poor joints.
- After the walls fails either due to bending or shearing in combination with the compressive loads, the whole houses crashes down.

- Extensive damage was observed during earthquake especially if it occurs after a rainfall. (Krishna + chandra – 1983)

How to improve mud + adobe building

- Better performance is obtained by mixing the mud with clay to provide the cohesive strength.
- The mixing of straw improves the tensile strength.
- coating the outter wall with water proof substance such as bitumen improves against weathering.
- The strength of mud walls can be improved significantly by split bamboo or timber reinforcement.
- Timber frame or horizontal timber runners at lintel level with vertical members at corners further improves its resistance to lateral forces which has been observed during earthquakes.

2) Masonry Building:-

- Masonry buildings of brick and stone are superior with respect to durability, fire resistance, heat resistance & formative effects.
- Masonry buildings consists of various material and sizes
  - (i) Large block (block size > 50 cm) – concrete blocks, rock blocks & lime stones
  - (ii) concrete brick – solid & hollow
  - (iii) Natural stone masonry
- They are easily available & economic reasons ~~used~~ and so this type construction used.
- In very remote areas in Himalayas building are constructed of stacks of random rock pieces without any mortar.

* Majority of new construction use mud mortar & few use cement mortar.

Causes of failure of masonry buildings :-

* These buildings are very heavy & attract large inertia forces.

* Unreinforced masonry walls are weak against tension (Horizontal forces) & shear.

* So masonry buildings are perform poor during earthqua

* Masonry buildings have large in plane rigidity & therefore have low time periods of vibration in large seismic force.

* Masonry buildings fall apart and collapsed because lack of integrity.

* Lack of integrity reasons → absense of bonding between cross walls, absense of diaphragm action of roofs & lack of box light action

Common type of damage in masonry building :-

* Severe damage resulting in complete collapse and pile up in a heap of stones.

* The inertia forces due to roof or floor is transmitted to the top of the walls.

* If the roofing material is improperly tied to the wall, it will be dislodged.

* The weak roof support connection is the cause of seperation of roof from the support and leads to complete collapse.

* The failure of bottom chord of roof truss may ~~be~~ also cause complete collapse of truss.

* If the roof/floor material is properly tied to the top walls causing it to shear of diagonally in the direction of motion through the bedding joints.

- The cracks usually intiate at the corners of the openings.

- The failure of pier occurs due to combined action of flexure and shear.

- vertical cracks near wall joint occur indicating separations of walls.

- For motion perpendicular to the walls, the bending moment at the ends result in cracking and seperation of the walls due to poor bonding.

- Due to high inertia force, masonry wall bulge outward or inward.

- The falling away of half the wall thickness on the bulged side is common features.

- The bonding stone is found to be effective as in Jammu kashmir earthquake of August 24, 1980.

- Unreinforced dressed rubble masonry (DRM) has slightly better performance than random rubble masonry.

- The most common damage is due to cracks in the walls.

- Masonry with lower unit mass and greater bond strength shows better performance.

- Un reinforced masonry should be avoided as construction material in seismic areas.

(3) **Reinforced Masonry Buildings:-**

- Reinforced masonry buildings have withstand earthquakes well, without appreciable damage.

- For horizontal bending, a tough member capable of taking bending, is found to perform better during earthquakes.

- If the corner sections or opening are reinforced wi steel bars even greater strength is attained.
- Dry packed stone masonry wall with continuous lint band over openings and cross walls did not undergo any damage.

4) Brick - Reinforced concrete frame buildings:-

- This type of building consists of RC frame structures and brick lay in cement mortar as infill.
- This type of construction is suitable in seismic areas.

Causes of failure of RC frame buildings:-

- The failures are due to mainly lack of good design of beams/columns frame action and foundation.
- Poor quality of construction inadequate detailing or laying of reinforcement in various components particularly at joints and in columns/beams for ductility.
- Inadequate diaphragm action of roof & floors.
- Inadequate treatment of masonry walls.

Common type of damage in RC frame buildings:-

- The damage is mostly due to failure of infill, or failure of columns or beams.
- Spalling of concrete in columns.
- Cracking or buckling due to excessive bending combined with dead load may damage the column.
- The buckling of columns are significant when the columns are slender and the spacing of stirrups in the column is large.

* Severe crack occurs near rigid joints of frame due to shearing action, it may lead to complete collapse.

* The differential settlement causes excessive moments in the frame and may lead to failure.

* Design of frame should be such that the plastic hinge is confined to beam only, because beam failure is less damaging than the common failure.

5) Wooden Buildings:-

* It is the most common type of construction in areas of higher seismicity.
* It is most suitable material for earthquake resistant construction due to its light weight and shear strength across the gains as observed in 1933, Long beach, 1952 Kern country, 1963 skopje and 1964 Anchorage earthquake.

* During Tokachi earthquake (1968), more than 4,000 wooden buildings were partially damaged.
* There were failure due to sliding and caving due to softness of ground.

* The main reasons of failure was its low rigidity joints, act as a hinge.

* Failure is also due to deterioration of wood with passage of time.

* Wood frames without walls have almost no resistant against horizontal forces.

* Resistant is highest for diagonal braced wall.
* Buildings with diagonal bracing in both vertical and horizontal plane perform much better.

- * The traditional wood frame Ikra construction of Assam and houses of Nicobars founded on wooden piles separated from ground have performed very well during earthquakes.

- * wood houses are generally suitable up to two ~~ways~~ storeys.

6) Reinforced concrete Buildings:-

- * This type of construction consists of shear walls and frames of concrete.
- * Substantial damage to reinforced concrete buildings was seen in the Kanto (1923) earthquake.
- * Later in Niigata (1964), off-Tokachi (1968) and Venezuela (1967) earthquake, it is suffered heavy damages.

- * The damage to RC buildings may be divided broadly into vibratory failure and tilting or uneven settlement.

- * A RC building is constructed on comparatively hard ground vibratory failure is seen.

- * on soft ground tilting, uneven settlement or sinking is observed.

- * Vibratory failure causes of ~~damaged~~ may be considered during earthquake
  - * exceeded the loads considered in design
  - * building did not have adequate resistance Strength
  - * ductility to withstand

- * Shear walls are found to be effective, to provide adequate Strength to the buildings

- * severe damage to spandrel wall b/w the vertical opening.

* Tilting and sinking ~~of~~ reinforced concrete building during earthquakes were seen in the Kanto & Niigata earthquake.

* Dead weights could not be supported after the settling of the ground.

* In soft ground, the damage becomes higher in the following order,
  - pile foundation
  - Mat foundation
  - continuous foundation
  - independent foundation.

* Hollow concrete block building with steel reinforcement in selected grout filled cells have shown good performance.

* Precast and prestressed RC buildings suffered severe damage mostly because of poor behaviour of joints & supports.

* Precast & prestressed element as a rule were not destroyed as observed in 1952 Kern Country and 1964 Anchorage earthquakes.

7) Steel skeleton Buildings:-

* Building with steel skeleton construction differ greatly according to shape of cross sections and method of connection.

* Divided into two varieties
  * braces as earthquake resistant elements
  * Rigid frame structures

* The former is used in low building, later used in high rise buildings.

* Braces → used earthquake resistant elements, it is normal to design, so that all horizontal forces will be borne by the braces.

* Generally this type of building is light and influence of wind loads is dominant in most case

* Many cases, the braces have breaking or bucklin so the joints have failed.

* The frames are comprised of beams and columns, consist of member H- beams is often used in high rise building.

* Non-structural damage is common but none of thes building severely damages as observed in 1906 San Francisco earthquake.

8) **Steel and reinforced concrete composite structures :-**

* Composed of steel skeleton & RC and have the dynamic characteristics of both.

* It is better with respect to fire resistance and saftey against buckling as compared to steel skelton.

* Compared to reinforced concrete structure it has better ductility after yielding.

* It has better earthquake resistant and to perform better during earthquakes.

9) **Prestressed concrete structures :-**

* PSC has long been accepted in statically loaded structures.

* For many years, seen the construction of PSC bridges, dams, runways, pipe lines, reservoirs and various structures including more recently atomic reactor pressure vessels.

- In recent years, PSC has been used in seismic resistant structures.
- Many thousands of structures has been constructed in PSC.
- Large frame structures are constructed in PSC and these have performed satisfactorily under normal Static and wind loading.

Design approach for framed buildings with Prestressed beams :-

- In framed buildings with prestressed beams and prestressed or normally reinforced columns.
- The prestressing of the beams has generally designed to resist gravity loading and the prestressing has been arranged that the vertical dead and seismic live loadings are balanced by the upward loading produced by post-tensioned tenons anchored in the column.
- The columns + beams are subjected to direct Compressive forces only and are able to resist the design lateral loadings applied in either directions.
- In some buildings, the post tensioning cables have been extended through the end columns and the anchorages for each beam encased in a concrete block.
- mortar joints are generally used b/w the elements.
- Precast frames assembled from units and post tensioned together have been assumed to be equivalent to similar cast in place frames.
- In some cases, Provision has been made to Prevent the mortar from dropping out of vertical mortar joints, should have large tension cracks occurs during Severe earthquake.

* To simple frames assembled by post-tensioning bea and columns, several flat slab structures have been built in which frame action is provided by moment transfer b/w columns and slabs.

* Both solid and waffle slabs have been used.

Shear wall buildings:-

* Concrete shear walls or shear cores with vertica post-tensioning have been used in a number of buildings.

* Post tensioning has bean considered to have advantages over normal mild steel reinforcing, it provides for full length reinforcing tendons without the need for splicing at points of high tensile stress congesting of reinforcing in Slender walls are reduced.

Behaviour of PSC under actual earthquakes:-

1) Skopje in 1963

* many buildings constructed in PSC structures.
* one of the shop buildings at new skopje steel work was a Precast, Prestressed concrete structures
* It consist of precast roof construction on continuous prestressed concrete columns.
* columns were I-shaped & pretensioned.
* The earthquake caused the building to rock on the columns in the direction of the girders.
* This induced severe bending moments at the top and bottom of each flange in the weak direction of bending, resulting in crushed concrete at the toes of the flanges.

- The Prestress in the flanges was destroyed, the columns could be readily repaired to restore their vertical load carrying capacity, the prestress could not be restored.

- The stiffness under lateral loadings would be reduced because of earlier cracking.

- ∴ auxiliary bracing system has to be provided, if the structure was have ability to resist horizontal forces in longitudinal direction.

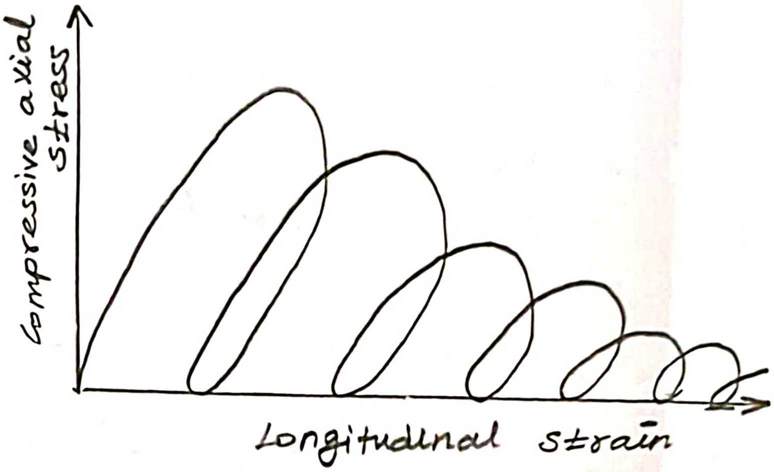
Cyclic behaviour of concrete and Reinforcement

Bouchinger & Pinching effects :-

1) Plain concrete

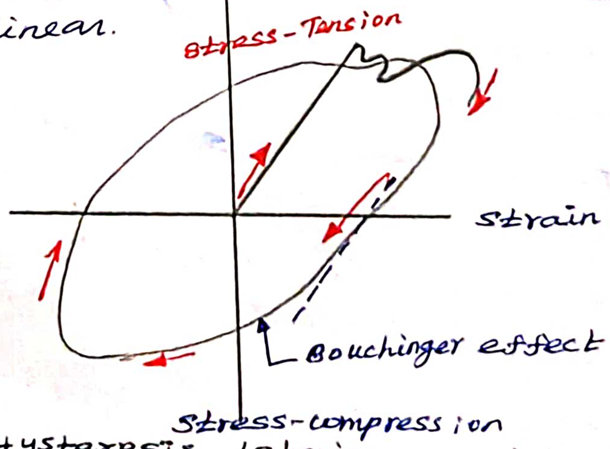
- plain concrete is a brittle material.
- During the first cycle, the stress strain curve is the same as that obtained from static test.
- If the specimen is unloaded and reloaded in compression, stress-strain curves similar to obtained.
- It can be seen that slope of the stress strain curves as well as maximum attainable stress decreases with the number of cycles.
- The stress strain relationship for plain concrete subjected to repeated compressive loads is cycle dependent.
- The decrease in stiffness and strength of plain concrete is due to the formation of cracks.
- The compressive strength of concrete depends on the rate of loading.

- The rate of loading increases, the compressive strength of concrete increases, but the strain at the maximum stress decreases.
- Plain concrete cannot be subjected to repeated tensile loads since its tensile strength is practically zero.



2) Reinforcement

- Reinforcing steel has much more ductility than plain concrete.
- The ultimate strain in mild steel is 25% , in concrete 0.3%
- In the first cycle, the reinforcing steel shows stress strain curve similar to that of obtained in the static test.
- After the specimen has reached its yield level and direction of load is reversed, ie, unloading begins, the loading curve is not straight but curvilinear.



Hysteresis behaviour of reinforcing steel

* This curvature in the unloading segment of stress-strain curve is raffered to the Bouchinger effect after the discoverer of the Penomenon.

* One complete cycle of loading and unloading is reffered to as a hysteresis loop.

* The area with in a hysteresis loop ~~exib~~ exhibits energy absorbed by the specimen in a cycle.

* In subsequent cycles practically the same path is repeated.

* The stress-strain relationship for mild reinforcing steel subjected to repeated reversed loading is cycle independent until the specimen buckles or fails due to fatigue.

* It is also observed that same hysteresis loops are obtained for a specimen which is first loaded in tension followed by compression as when it is first loaded compression followed by tension.

* The yield strength of reinforcement is alsa affected by the rate of loading.

Evalauation of earthquake forces :- IS : 1893 - 2002

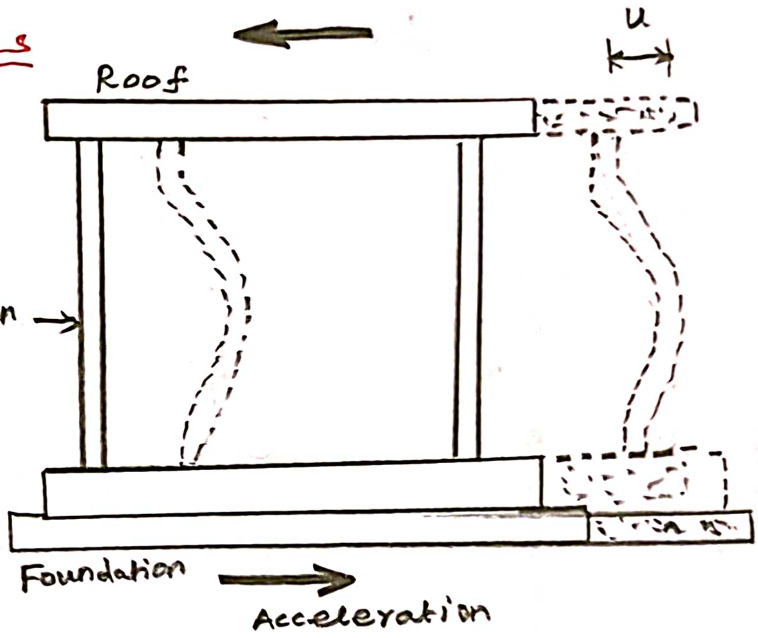
* Recommendations provided by seismic codes, help the designer to improve the behaviour of structure

*

* to withstand the earthquake effects without significant loss.

*

Inertia forces in structures



* Earthquake causes shaking of the ground.

* building resting on it will experience motion at its base

* The base of the building moves with the ground the roof has a tendency to stay in its original Position.

* But the walls and columns are connected to it, they drag the roof along with them.

(The situation, when the bus you are standing is suddenly starts; your feet move with the bus, but your upper body tends to stay back making you fall backwards)

* This tendency to continue to remain in the Previous position is known as inertia.

* In the building since the walls or columns are flexible, the motion of the roof truss is different from that of the ground.

* When the ground moves, even the building is thrown backwards, and the roof experiences a force called inertia force.

Response spectrum :- IS : 1893 - 2002

* Response spectrum are curves plotted b/w maximum response of SDF system subjected to specified earthquake ground motion and its time period.

* Response spectrum can be interpreted as the locus of maximum response of a SDF system for given damping ratio.

* It can be used for obtaining lateral forces developed in structures due to earthquake.

* Response spectral values depends upon the following parameters

- Energy release mechanism
- Epicentral distance
- focul depth
- Soil condition
- Richter magnitude
- Damping in the system
- Time period of the system.

* Response spectrum is a plot of max. response, namely

- Max displacement
- Velocity
- acceleration

* In plot, X-axis denotes time period or natural frequency.

Y-axis denotes displacement or velocity or acceleration

* Each of response spectrum consider a specific value of damping ratio (ϱ)

* several such response spectrum must be developed to cover a entire range of damping ratio encountered in actual structures.

* Response spectrum is usually named after choosen peak response quantity.

(i) Relative display spectrum

(ii) Relative velocity spectrum

(iii) Relative acceleration spectrum

Explain in

Unit IV - Earthquake Load Analysis.

5 (b)(i)

Step by step procedure for seismic Analysis of RC Buildings

1. Determination of natural period of vibration
Fundamental natural period.
* Based on infill panels.

* Moment resisting frame with out infill panels

IS:1893 pt(1) P.NO: 24

Fundamental natural period

$$\boxed{T_a = 0.075 h^{0.75}}$$

h → ht of building in 'm'
d → Base dimension of building at plinth level in 'm'

* Moment resisting frame with infill panels

$$\boxed{T_a = \frac{0.09 h}{\sqrt{d}}}$$

2) Determination of other important factors.

P.NO:16 Fig (2) → (i) $\frac{S_a}{g}$ → Average response acceleration co-efficient

P.NO:35 → (ii) Z → Zone factor

(iii) I → Important factor

P.NO: 18 → (iv) R → Response Reduction factor

3) Determination of horizontal seismic co-efficient

$$\boxed{A_h = \frac{Z I}{2R}\left(\frac{S_a}{g}\right)}$$

4) Determination of design vertical seismic co-efficient

Design Vertical seismic co-efficient = 2/3 design horizontal seismic co-efficient

5) Determination of design base shear

$$\boxed{V_B = A_h \cdot W}$$

A_h → design horizontal Se co-efficient

W → Seismic Wt of the building.

6) Distribution of equivalent lateral load

* design base shear computed shall be distributed alo the height of the building,

$$\boxed{Q_i = V_B \left[\frac{W_i h_i^2}{\sum_{j=1}^{n} W_j h_j^2} \right]}$$

Q_i → Design lateral force at floor i

W_i → seismic wt of floor i,

h_i → ht of floor i measured from base

n → number of storeys in the building is the number of levels at which the masses are located.

h_i → calculated from base

$$Q_1 = V_B \left[\frac{w_1 h_1^2}{w_1 h_1^2 + w_2 h_2^2 + w_3 h_3^2} \right]$$

---

15 a) Determine the design horizontal seismic co-efficient for an ordinary reinforced concrete moment resisting frame hospital building without infill panels for a damping of 5% The building is situated in salem. Height of the building is 22m and it is resting on hard soil.

Step: 1 - Determination of natural period of Vibration

For → RC moment resisting frame without brick panels

$$\therefore T_a = 0.075 h^{0.75} = 0.075 \times (22)^{0.75}$$

$$\boxed{T_a = 0.76 \text{ sec}}$$

1893(1)-2002
P.NO: 16
Fig: 2

Step: 2 - Determination of Other important factors:

$T_a = 0.76$ sec

$\left(\frac{S_a}{g}\right) \rightarrow 1.35$

S:1893(1)-2002
P.NO: 35

Zone factor (Z)

Salem, Zone - III, Hospital building.

$$\boxed{Z = 0.16}$$

Importance factor (I) Hospital building

$$\boxed{I = 1.5}$$

P.NO: 18

Response reduction factor (R)

For ordinary moment resisting frame,

$$\boxed{R = 3}$$

Step: 3 - Determination of design horizontal seismic co-efficient (A_h)

$$A_h = \frac{Z\,I}{2R}\left(\frac{S_a}{g}\right) = \frac{0.16 \times 1.5 \times 1.35}{2 \times 3}$$

$$\boxed{A_h = 0.054}$$

---

15 b A special reinforced concrete moment resisting frame building with infill panels is situated in Delhi. Height of the building is $\overset{h}{12}$ m. The building is resting on medium soil. The base dimensions of building at plinth level is $\overset{d}{24}$ m. Determine the design horizontal seismic co-efficient and vertical seismic co-efficient for a damping of 2%.

Step: 1 - Natural period of vibration

For RC special moment resisting frame with infill pane

$$T_a = \frac{0.09h}{\sqrt{d}} = \frac{0.09 \times 12}{\sqrt{2.4}} = 0.22 \text{ sec.}$$

reinforced concrete scha

Step: 2 - Determination of other important factors

(i) Average Response acceleration co-efficient $\left(\frac{S_a}{g}\right)$

P.NO: 16
Table:

2% damping, multiplying factor = 1.4

P.NO: 16
Fig: 2

$$\frac{S_a}{g} = 2.5$$

∴ for 2% damping, $\frac{S_a}{g} = 1.4 \times 2.5 = 3.5$

$$\boxed{\frac{S_a}{g} = 3.5}$$

(ii) Zone factor (Z)

Delhi, Zone: IV, $\boxed{Z = 0.24}$

(iii) Importance factor (I)

For general structure, $\boxed{I = 1}$

(iv) Response Reduction factor (R)

for, special moment resisting frame building,

$$\boxed{R = 5}$$

Step: 3 - Design horizontal Seismic co-efficient (A_h)

$$A_h = \frac{Z\,I}{2R}\left(\frac{S_a}{g}\right) = \frac{0.24 \times 1 \times 3.5}{2 \times 5}$$

$$\boxed{A_h = 0.084}$$

Step: 4 - Design Vertical Seismic co-efficient (V_c)

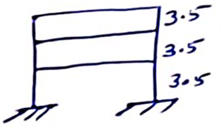
1893
P.N
Fi

$$V_c = \frac{2}{3} A_h = \frac{2}{3} \times 0.084$$

$$\boxed{V_c = 0.056}$$

A three storeyed, symmetrical reinforced concrete school building in zone V with plan dimensions $\overset{d}{7m}$, storey height of 3.5m, Total weight of beams in a storey is 130KN and total weight of slab in a storey 250KN. Total weight of columns in a storey is 50KN and total weight of walls in the storey is 530KN. Live load = 130KN, weight of terrace floor is 655KN. resting on hard ~~soil~~ rock, damping = 5%. Determine the base shear and lateral loads at each floor by seismic co-efficient method.

$h = \overset{\text{storey}}{3} \times \overset{\text{ht}}{3.5} = 10.5m$



Step: 1

P.NO: 25

$$T_a = \frac{0.09h}{\sqrt{d}} = \frac{0.09 \times 10.5}{\sqrt{7}} = 0.36\,\text{sec}$$

Step: 2

(i) $\frac{S_a}{g} \rightarrow$ for hard ~~soil~~ rock, $T = 0.36$

$0.1 \leq T \leq 0.4$

$$\frac{S_a}{g} = 2.5$$

Damping 5%.

∴ IS: 1893 (Pt: 1) - 2002 ; P.NO: 17, Table: 3

multiplying factor 5% = 1

$$\therefore \boxed{\frac{S_a}{g} = 2.5}$$

P.NO: 16 (ii) Zone V $\rightarrow \boxed{Z = 0.36}$

(iii) Importance factor $\boxed{(I) \rightarrow 1.5}$ for school building

P.NO: 18

P.NO: 23 (iv) Reduction factor (R)

Take SMRF
symmetrical reinforced

$\boxed{R = 5}$

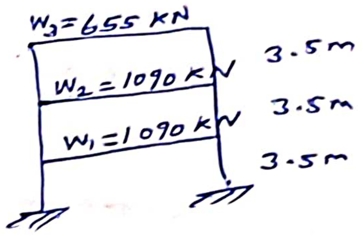
Step: 3

$$A_h = \frac{ZI}{2R}\left(\frac{S_a}{g}\right) = 0.135$$

Seismic weight

Beam	wt	= 130 KN
Slab	wt	= 250 KN
Column	wt	= 50 KN
Wall	wt	= 530 KN
	L.L	= 130 KN
		1090 KN

Terrace wt = 655 KN



Total Seismic wt of building (W) = $W_1 + W_2 + W_3$

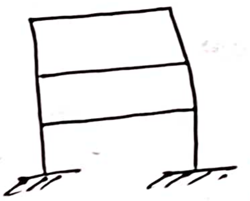
$= 1090 + 1090 + 655$

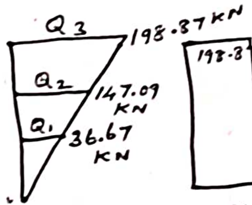
$$\boxed{W = 2835 \text{ KN}}$$

step: 4

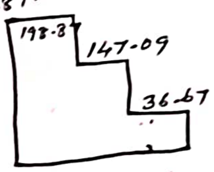
$V_B = A_h \cdot W = 0.135 \times 2835$ KN

$$\boxed{V_B = 382.73 \text{ KN}}$$





force diagram



$V_B = 382.73$ KN

shear distribution

step: 5

$$Q_1 = V_B \left[\frac{W_1 h_1^2}{W_1 h_1^2 + W_2 h_2^2 + W_3 h_3^2} \right]$$

$$= 382.73 \left[\frac{1090 \times 3.5^2}{1090 \times 3.5^2 + 1090 \times 7^2 + 655 \times 10.5^2} \right]$$

138976.25

$$\boxed{Q_1 = 36.77 \text{ KN}}$$

$$Q_2 = 382.73 \left[\frac{1090 \times 7^2}{1090 \times 3.5^2 + 1090 \times 7^2 + 655 \times 10.5^2} \right]$$

$$\boxed{Q_2 = 147.09 \text{ KN}}$$

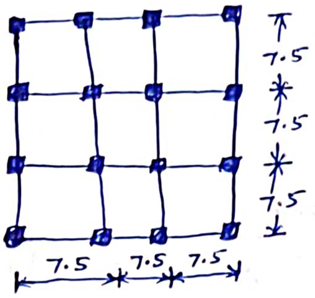
$$Q_3 = 382.73 \left[\frac{655 \times 10.5^2}{1090 \times 3.5^2 + 1090 \times 7^2 + 655 \times 10.5^2} \right]$$

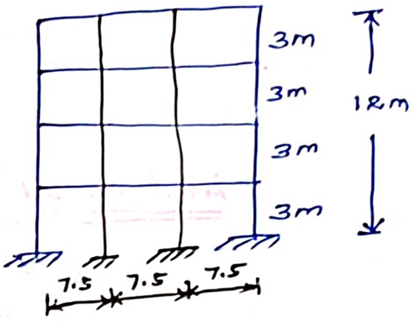
$$\boxed{Q_3 = 198.87 \text{ KN}}$$

Check

$V_B = Q_1 + Q_2 + Q_3 = 382.73$ KN. Hence ok.

Plan and elevation of a four storey reinforced concrete office building as shown in fig. The details of the building are as follows.





Zone - III

Live load = 3 kN/m^2

Columns = 450×450 mm

Beams = 250×400mm

Thickness of slab = 150mm

Thickness of wall = 120mm

Importance factor = 1

Number of storey = 4

Structure type = OMRF building.

Determine design seismic lateral load & storey shear force distribution.

Step: 1 – computation of seismic weight

Assume, unit wt of concrete = 25 kN/m^3

unit wt of brick = 20 kN/m^3

Slab

D.L. (or) self wt of slab = $(L\,B\,D)\,\gamma_c = \overset{L}{(7.5+7.5+7.5)} \times \overset{B}{(7.5+7.5+7.5)} \times \overset{D}{0.15} \times \overset{\gamma_c}{25}$

$$\boxed{DL = 1898.4 \text{ kN}}$$

Beam

Self wt of beam $= L.B.D\ \gamma_c = \underbrace{(\overset{\text{No. of beam}}{24} \times \overset{\text{length}}{7.5})}_{180\text{ m}} \times \overset{B}{0.25} \times \overset{D}{0.4} \times \overset{\gamma_c}{25}$

$= 450$ kN

Columns

Self wt. of column $= \text{No. of columns} \times \overset{ht}{L}.B.D.\gamma_c$

$= 16 \times \overset{ht}{3\text{m}} \times \overset{b}{0.45} \times \overset{d}{0.45} \times \overset{\gamma_c}{25}$

$= 243$ kN.

Walls

Self wt of wall = ~~length × ht ×~~ $LBD\ \gamma_b$

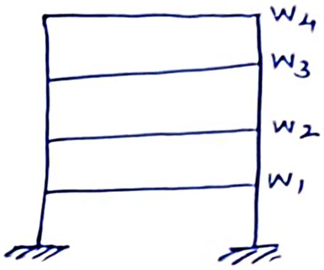
$= (\overset{\text{No. of panel}}{24} \times \overset{\text{length}}{7.5}) \times \overset{\text{thick}}{0.12} \times \overset{ht}{3} \times \overset{\gamma_b}{20} = 648$ kN

Live Load (or) Imposed load on slab

$$\underset{\text{on slab}}{L.L} = L.B.H \times 25\% = \overset{\text{Length}}{(7.5+7.5+7.5)} \times \overset{\text{breadth}}{(7.5+7.5+7.5)} \times \overset{\text{ht}}{3} \times \frac{25}{100}$$

$$L.L = 380\ kN$$

Load on all floors



$$W_1 = \overset{D.L}{(\text{Slab} + \text{Beam} + \text{column} + \text{Wall})} + L.L$$

$$= \overset{\text{slab}}{1898.4} + \overset{\text{slab L.L}}{380} + \overset{\text{beam}}{450} + \overset{\text{column}}{243} + \overset{\text{wall}}{648}$$

$$\boxed{W_1 = 3619\ kN}$$

|||ly $\boxed{W_1 = W_2 = W_3 = 3619\ kN}$

Load on roof slab ~~(Terrace)~~

$$W_4 = \overset{D.L}{\text{slab}} + \underset{\text{zero}}{\cancel{L.L_{\text{slab}}}} + \underset{\text{Beam}}{DL} + \frac{\underset{\text{column}}{DL}}{2} + \frac{\underset{\text{Wall}}{DL}}{2}$$

$$= 1898.4 + 0 + 450 + \frac{243}{2} + \frac{648}{2}$$

$$\boxed{W_4 = 2793.9\ kN}$$

∴ Total seismic weight $W = W_1 + W_2 + W_3 + W_4$

$$\boxed{W = 13650.9\ kN}$$

Step: 2 - Natural period

consider stiffness of Infill masonry

$$\therefore T_a = \frac{0.09h}{\sqrt{d}} = \frac{0.09 \times 12}{\sqrt{22.5}}$$

$h = 12m$

base dimension $d = 7.5+7.5+7.5$

$d = 22.5m$

$$\boxed{T_a = 0.228\ sec}$$

Step: 3

Average response acceleration co-efficient

(i) $\frac{S_a}{g} \rightarrow$ Average response

$T_a = 0.228$ sec ; Type of soil : medium

$$\boxed{\frac{S_a}{g} = 2.5}$$

Plan and elevation of a four storey reinforced concrete office building as shown in fig. The details of the building are as follows.

Zone - III

Live load = 3 kN/m^2

Columns = 450 × 450 mm

Beams = 250 × 400 mm

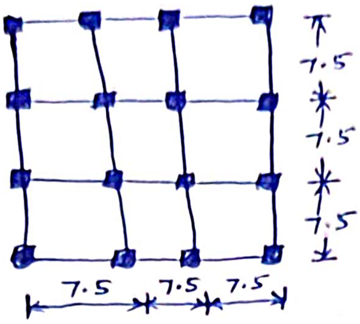
Thickness of slab = 150 mm

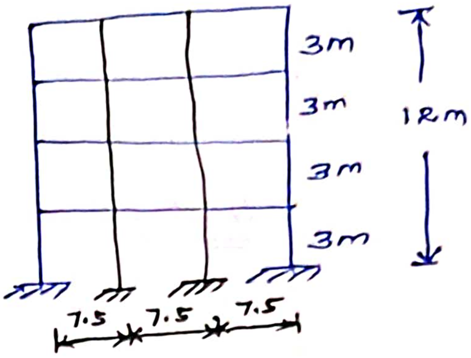
Thickness of wall = 120 mm

Importance factor = 1

Number of storey = 4

Structure type = OMRF building.





Determine design seismic lateral load & storey shear force distribution.

Step: 1 - computation of seismic weight

Assume, unit wt of concrete = 25 kN/m^3

unit wt of brick = 20 kN/m^3

Slab

$$D.L.\ (\text{or})\ \text{self wt of slab} = (L\,B\,D)\,\gamma_c = \overset{L}{(7.5+7.5+7.5)} \times \overset{B}{(7.5+7.5+7.5)} \times \overset{D}{0.15} \times \overset{\gamma_c}{25}$$

$$\boxed{DL = 1898.4\ kN}$$

Beam

$$\text{self wt of beam} = L.B.D\ \gamma_c = \underbrace{(\overset{\text{No. of beam}}{24} \times \overset{\text{length}}{7.5})}_{180\ m} \times \overset{B}{0.25} \times \overset{D}{0.4} \times \overset{\gamma_c}{25}$$

$$= 450\ kN$$

Columns

$$\text{Self wt. of column} = \text{No. of columns} \times \overset{ht}{L}.B.D.\gamma_c$$

$$= 16 \times \overset{ht}{3m} \times \overset{b}{0.45} \times \overset{d}{0.45} \times \overset{\gamma_c}{25}$$

$$= 243\ kN.$$

Walls

Self wt of wall = ~~length × ht ×~~ $LBD\ \gamma_b$

$$= (\overset{\text{No. of panel}}{24} \times \overset{\text{length}}{7.5}) \times \overset{\text{thick}}{0.12} \times \overset{ht}{3} \times \overset{\gamma_b}{20} = 648\ kN$$

Live Load (or) Imposed load on slab

$$\underset{\text{on slab}}{L.L} = L.B.H \times 25\% = \overset{\text{Length}}{(7.5+7.5+7.5)} \times \overset{\text{breadth}}{(7.5+7.5+7.5)} \times \overset{\text{ht}}{3} \times \frac{25}{100}$$

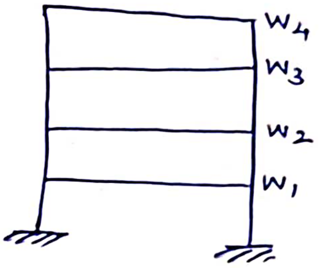
$$L.L = 380 \text{ kN}$$

Load on all floors

$$W_1 = \overset{DL}{(\text{slab} + \text{Beam} + \text{column} + \text{wall})} + L.L$$

$$= \overset{\text{slab}}{1898.4} + \overset{\text{slab L.L}}{380} + \overset{\text{beam}}{450} + \overset{\text{column}}{243} + \overset{\text{wall}}{648}$$

$$\boxed{W_1 = 3619 \text{ kN}}$$



$|||^{ly}$

$$\boxed{W_1 = W_2 = W_3 = 3619 \text{ kN}}$$

Load on roof slab (~~Terrace~~)

$$W_4 = \overset{D.L}{\text{slab}} + \underset{\text{zero}}{\cancel{\underset{\text{slab}}{L.L}}} + \underset{\text{Beam}}{DL} + \frac{\underset{\text{column}}{DL}}{2} + \frac{\underset{\text{Wall}}{DL.}}{2}$$

$$= 1898.4 + 0 + 450 + \frac{243}{2} + \frac{648}{2}$$

$$\boxed{W_4 = 2793.9 \text{ kN}}$$

∴ Total seismic weight $W = W_1 + W_2 + W_3 + W_4$

$$\boxed{W = 13650.9 \text{ kN}}$$

Step: 2 - Natural period

consider stiffness of Infill masonry

$$\therefore \quad T_a = \frac{0.09h}{\sqrt{d}} = \frac{0.09 \times 12}{\sqrt{22.5}}$$

$h = 12m$

base dimension $d = 7.5+7.5+7.5$

$d = 22.5m$

$$\boxed{T_a = 0.228 \text{ sec}}$$

Step: 3

(i) $\frac{S_a}{g}$ → Average response acceleration co-efficient

$T_a = 0.228$ sec ; Type of soil : medium

$$\boxed{\frac{S_a}{g} = 2.5}$$

(ii) Zone factor (Z) → zone - III

$$\boxed{Z = 0.16}$$

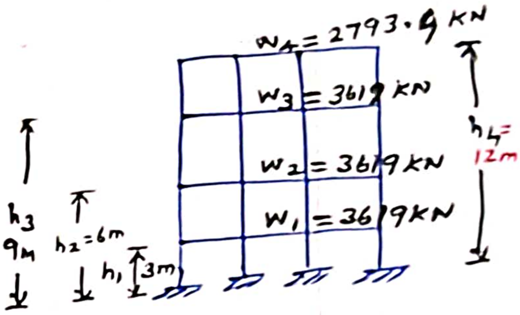
(iii) Importance factor $\boxed{I = 1}$ (given

(iv) Response Reduction factor $\boxed{(R) = 3}$ (OMRF)

Step: 4 - Horizontal seismic co-efficient (A_h)

$$A_h = \frac{ZI}{2R}\left(\frac{S_a}{g}\right) = \frac{0.16 \times 2.5 \times 1}{2 \times 3} = 0.0667$$

Step: 5 - Base shear



$$V_B = A_h . W = 0.0667 \times 13650.9$$

$$\boxed{V_B = 910.033 \text{ KN}}$$

Step: 6 - Lateral forces

$$Q_1 = V_B \left[\frac{W_1 h_1^2}{W_1 h_1^2 + W_2 h_2^2 + W_3 h_3^2 + W_4 h_4^2}\right] = \frac{(3619 \times 3^2) \times 910.033}{[(3619 \times 3^2) + (3619 \times 6^2) + (3619 \times 9^2) + 2793.9 \times 12^2)]}$$

$Q_1 =$

$$Q_2 = \frac{(3619 \times 6^2) \times 910.033}{[(3619 \times 3^2) + (3619 \times}$$

Response Spectrum method

14 (b) Determine the design lateral forces at each floor for a two storey RC shear frame of a hospital building for the following data. Use response spectrum method of IS 1893-2002

April-May 2018

Seismic weight of each floor = 50 kN

spacing b/w columns = 3 m c/c

Height of each floor = 3 m

Type of structure = special moment resisting frame

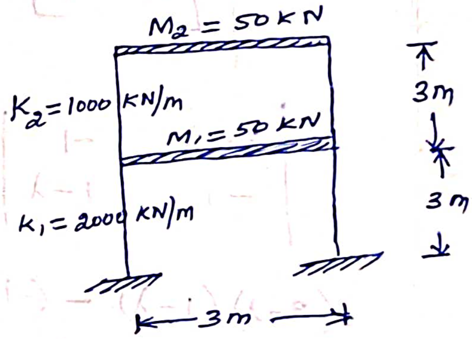
Location of the building = coimbatore

Type of soil = Rock

combined stiffness of ground floor columns : 2000 kN/m

combined stiffness of first floor columns : 1000 kN/m

Solution :-

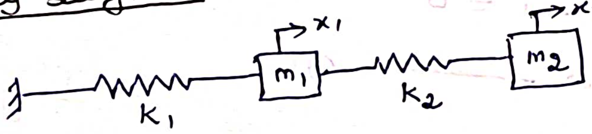


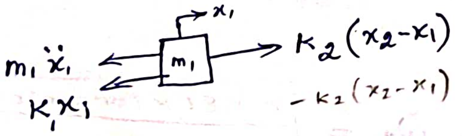
$$\text{Mass} = 50\text{ kN} = 50{,}000\text{ N}$$

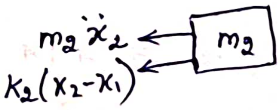
$$= \frac{50{,}000}{9.81}\text{ kg}$$

$$\boxed{m_1 = m_2 = 5096.84\text{ kg}}$$

Free Body diagram







$$m_1\ddot{x}_1 + K_1x_1 - K_2x_2 + K_2x_1 = 0$$

$$m_1\ddot{x}_1 + K_1x_1 + K_2x_1 - K_2x_2 = 0$$

$$m_1\ddot{x}_1 + (K_1+K_2)x_1 - K_2x_2 = 0 \quad ①$$

$$m_2\ddot{x}_2 + K_2(x_2-x_1) = 0$$

$$m_2\ddot{x}_2 + K_2x_2 - K_2x_1 = 0$$

$$m_2\ddot{x}_2 - K_2x_1 + K_2x_2 = 0 \quad ②$$

Mass matrix

$$[m] = \begin{bmatrix} m_1 & 0 \\ 0 & m_2 \end{bmatrix} ;$$

$$[m] = \begin{bmatrix} 5096.84 & 0 \\ 0 & 5096.84 \end{bmatrix}$$

stiffness matrix

$$[K] = \begin{bmatrix} K_1+K_2 & -K_2 \\ -K_2 & K_2 \end{bmatrix}$$

$$[K] = \begin{bmatrix} 3000\times10^3 & -1000\times10^3 \\ -1000\times10^3 & 1000\times10^3 \end{bmatrix}$$

Step: 1 – Natural frequency (ω_1 & ω_2)

The general equation is

$$\left|[K] - \omega^2[M]\right| = 0$$

$$10^3 \times 1000 \left| \begin{bmatrix} 3 & -1 \\ -1 & 1 \end{bmatrix} - \omega^2\, 5096.84 \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \right| = 0$$

÷ by 1000×10^3

$$\left| \begin{bmatrix} 3 & -1 \\ -1 & 1 \end{bmatrix} - \frac{5096.84\,\omega^2}{1000 \times 10^3} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \right| = 0$$

$$\boxed{\lambda = \frac{5096.84\,\omega^2}{1000 \times 10^3}}$$

$$\left| \begin{bmatrix} 3 & -1 \\ -1 & 1 \end{bmatrix} - \begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix} \right| = 0$$

$$\begin{vmatrix} 3-\lambda & -1 \\ -1 & 1-\lambda \end{vmatrix} = 0$$

$$(3-\lambda)(1-\lambda) - (-1)(-1) = 0$$

$$3 - 3\lambda - \lambda + \lambda^2 - 1 = 0$$

$$\lambda^2 - 4\lambda - 2 = 0 \text{ ——— } ③$$

$$\lambda_1 = 3.414$$

$$\lambda_1 = \frac{5096.84\,\omega^2}{1000 \times 1000} = 3.414$$

$$\therefore \omega^2 = \frac{3.414 \times 1000 \times 1000}{5096.84}$$

$$\omega^2 = 669.83$$

$$\boxed{\omega_1 = 25.88 \text{ rad/sec}}$$

$$\lambda_2 = 0.586$$

$$\omega_2 = \sqrt{\frac{0.586 \times 1000 \times 1000}{5096.84}}$$

$$\boxed{\omega_2 = 10.72 \text{ rad/sec}}$$

Step: 2 – Natural Period (T)

$$T = \frac{2\pi}{\omega} \quad ; \quad T_1 = \frac{2\pi}{\omega_1} = \frac{2\pi}{25.88} = 0.24 \text{ sec}$$

$$T_2 = \frac{2\pi}{\omega_2} = \frac{2\pi}{10.72} = 0.58 \text{ sec}$$

Modal participation factor (P_K)

IS:1893-2002 ; P.NO: 26, cl: 7.8.4.5 (b)

Mode 1
$$\underline{P_{K_1}} = \frac{\sum_{i=1}^{n} W_i \phi_{ik}}{\sum_{i=1}^{n} W_i (\phi_{ik})^2} = \frac{29.3}{58.57} = \underline{0.50}$$

$$\underline{P_{K_2}} = \frac{170.7}{341.37} = \underline{0.50}$$

Step: 5 - Design lateral force

IS:1893-2002 ; P.NO: 26 ; cl : 7.8.4.5 (c)

$$Q_{ik} = A_K \cdot \phi_{ik} \, P_K \cdot W_i$$

Fore Mode 1

$$\boxed{T_1 = 0.24 \text{ sec}}$$

IS:1893-2002 P.NO: 16

$\left(\frac{S_a}{g}\right)_1 =$ for rock $(0.1 \leq T \leq 0.4)$

[illegible]

$$\boxed{\left(\frac{S_a}{g}\right) = 2.5}$$

$A_K = A_h$

P.NO: 14

$$A_h = \frac{Z I}{2R} \left(\frac{S_a}{g}\right)$$

$\boxed{Z = 0.16}$ (place: Coimbatore zone - III) P.NO: 35

$\boxed{I = 1.5}$ (hospital building) P.NO: 18 table: 6

$R = 5.0$ (SMRF) P.NO: 23, Table: 7

[illegible] 1.024

$$A_{h_1} = \frac{0.16 \times 1.5}{2 \times 5} \times 2.5$$

$$\boxed{A_{h_1} = 0.06}$$

For Mode 2

$$\boxed{T_2 = 0.58 \text{ sec}}$$

$(0.4 \leq T \leq 4)$

$$\left(\frac{S_a}{g}\right)_2 = \frac{1}{T} = \frac{1}{0.58}$$

$$\boxed{\left(\frac{S_a}{g}\right) = 1.72}$$

$$A_{h_2} = \frac{0.16 \times 1.5}{2 \times 5} \times 1.72$$

$$\boxed{A_{h_2} = 0.041}$$

step:3 - Mode shape

$$\left|[k]-\omega^2[M]\right|\{x\}=0$$

$$\begin{vmatrix} 3-\lambda & -1 \\ -1 & 1-\lambda \end{vmatrix} \begin{Bmatrix} x_1 \\ x_2 \end{Bmatrix} = 0$$

First mode

$\lambda_1 = 3.414$

$$\begin{vmatrix} \overset{-0.414}{(3-3.414)} & -1 \\ -1 & \overset{-2.414}{(1-3.414)} \end{vmatrix} \begin{Bmatrix} x_1 \\ x_2 \end{Bmatrix} = 0$$

$-0.414x_1 - x_2 = 0$

Assume $x_1 = 1$

$-0.414 = x_2$

$$\boxed{\phi_1 = \begin{Bmatrix} 1 \\ -0.414 \end{Bmatrix}}$$

second mode

$\lambda_2 = 0.586$

$$\begin{vmatrix} \overset{2.414}{(3-0.586)} & -1 \\ -1 & \overset{0.414}{(1-0.586)} \end{vmatrix} \begin{Bmatrix} x_1 \\ x_2 \end{Bmatrix} = 0$$

$2.414x_1 - x_2 = 0$

Assume $x_1 = 1$

$2.414 = x_2$

$$\boxed{\phi_2 = \begin{Bmatrix} 1 \\ 2.414 \end{Bmatrix}}$$

step: 4 - Model mass (M_k) & Modal participation factor (P_k)

Storey Level	Weight (W) kN	Mode 1			Mode 2		
		ϕ_1	$W.\phi_1$	$W.\phi_1^2$	ϕ_2	$W.\phi_2$	$W.\phi_2^2$
First floor (2)	50	1	50	(50×1^2) 50	1	50	50
Ground Floor (1)	50	-0.414	-20.7	$[50\times(0.414)^2]$ 8.57	2.414	120.7	291.37
	Total		29.3	58.57	-	170.7	341.37

IS: 1893 — 2002
P-NO: 26
Cl: 7.8.4.5 (a)

Model mass @ Mode 1, $M_{k_1} = \dfrac{[\Sigma W\phi_i]^2}{\Sigma W\phi_i^2} = \dfrac{(29.3)^2}{58.57} = 14.66\ kg$

@ Mode 2, $M_{k_2} = \dfrac{(170.7)^2}{341.37} = 85.36\ kg$

$Q_1 = A_{h_1} \cdot \phi_{k_1} P_{k_1} \cdot W_1$

~~1/0.06x~~

For first mode

$$\phi_1 = \begin{Bmatrix} 1 \\ -0.414 \end{Bmatrix} \begin{matrix} x_1 \\ x_2 \end{matrix}$$

$A_h = 0.06$

$P_{k_1} = 0.5$

$W_1 = 50 \text{ kN}$

Lateral force at first mode

$Q_{1-x_1} = A_{h_1} \cdot \phi_{1x_1} \cdot P_{k_1} \cdot W_1$

$= 0.06 \times 1 \times 0.5 \times 50$

$\boxed{Q_{1-x_1} = 1.5 \text{ kN}}$

$Q_{1-x_2} = A_{h_1} \cdot \phi_{1x_2} P_{k_1} \cdot W_1$

$= 0.06 \times (-0.414) \times 0.5 \times 50$

$\boxed{Q_{1-x_2} = -0.621 \text{ kN}}$

$Q_2 = A_{h_2} \cdot \phi_{k_2} \cdot P_{k_2} \cdot W_2$

~~0.041x~~

for second mode

$$\phi_2 = \begin{Bmatrix} 1 \\ 2.414 \end{Bmatrix} \begin{matrix} x_1 \\ x_2 \end{matrix}$$

Lateral force at 2nd mode

$Q_{2-x_1} = A_{h_2} \cdot \phi_{2x_1} P_{k_2} W_2$

$= 0.041 \times 1 \times 0.5 \times 50$

$\boxed{Q_{2-x_1} = 1.025 \text{ kN}}$

$Q_{2-x_2} = Ah_2 \cdot \phi_{2x_2} P_{k_2} W_2$

$= 0.041 \times 2.414 \times 0.5 \times 50$

$\boxed{Q_{2-x_2} = 2.474 \text{ kN}}$

Lateral force ~~Q~~

Floor level	Weight (W) kN	Mode 1			Second Mode		
		ϕ	Q_i	V_i	ϕ	Q_i	V_i
First floor 2	50	1	1.5	1.5	1	1.025	1.025
Ground floor 1	50	-0.414	-0.621	(1.5-0.621) 0.879	2.414	2.474	(1.025+2.47 3.499

Base shear

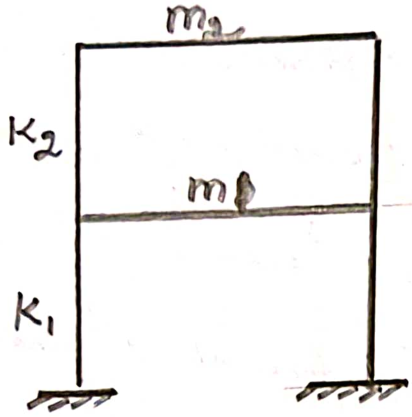
$V_{1\,(G.F)} = \sqrt{V_{i_1}^2 + V_{i_2}^2} = \sqrt{(3.499)^2 + (0.879)^2} = 3.61$ ~~[illegible]~~ kN

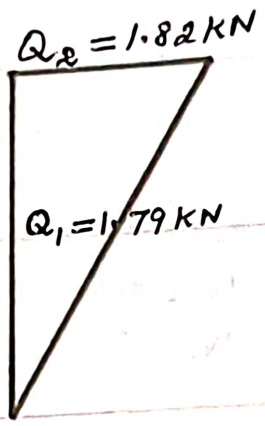
$V_2 = \sqrt{V_{i_2}^2 + V_{i_2}^2} = \sqrt{1.5^2 + 1.025^2} = 1.82 \text{ kN}$

Lateral load

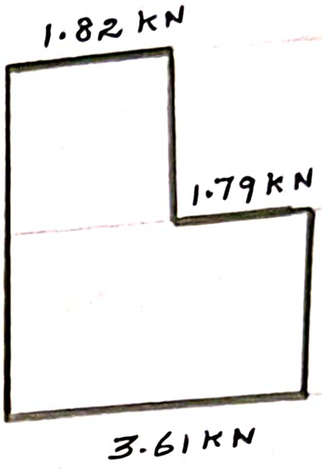
$Q_2 = V_2 = 1.82 \text{ kN}$

$Q_1 = V_1 - V_2 = 3.61 - 1.82 = 1.79 \text{ kN}.$





Lateral load



Base shear

Unit - V - Earthquake Resistant Design

Design Methodology

Load factors for plastic design of steel structures :-

combinations of (i) 1.7 (DL + IL)

(ii) 1.7 (D.L + EL) (iii) 1.3 (D.L + IL + EL)

Methods ~~of~~ improving element level ductility :-

- * Decreasing the tension steel area, yield stress & strain of the tension steel.
- * Increasing the ultimate compressive strain of Concrete.
- * Increasing the area of compression steel.
- * Reduction in the axial compression on the section.
- * Provision of effective confinement stirrups, hoops or ties. (compressive steel does not buckle).

IS : 13920 - Provisions for flexural members :-

- * Provisions apply to frame members resisting earthquake induced forces & designed to resist flexure.
- * These members shall satisfy the following provisions.

(i) Factored axial stress on the member under earthquake loading shall not exceed $0.1 f_{ck}$.

(ii) The members shall preferable have a width to depth ratio more than 0.3.

(iii) Width of the member shall not be less than 200mm

(iv) The depth (D) of the member shall preferably be not more than $\frac{1}{4}$ of clear span.

Review of IS 1893 - 2002 :-

- * The structures with stand without structural damage, moderate earthquakes and withstand without total collapse, heavy earthquakes.

* IS:1893 specifies various methods of analysis is
  - (i) seismic co-efficient (or) Equivalent lateral force met
  - (ii) Model analysis (or) Response spectrum metho

Mechanism of Base isolation:-

* It is subjected to ground motion.
* The isolation reduces the fundamental lateral freque of the structure from its fixed base frequenc & thus shifts the position of structure in th spectrum from peak plateau region.
* Also it brings forth additional damping due to the increased damping introduced at the base level & ~~this~~ thus reduction in the spectral acceleration is achieved.

Steps to improve Global level ductility:-

* Increasing the redundancy of the structure.
* Weak beam and strong column approach
* Avoiding soft first storey effects.
* Avoiding Non-ductile failure modes like shear, bond & axial compression at the element level.

Lateral load analysis of building system:-

* Earthquake force is an inertia force which is equal to mass times acceleration.
* Mass of the building is mainly located at its floors.
* Transferring the horizontal component of seismic force safely to the ground is the major task in seismic design.

* The floors should transfer the horizontal force to Vertical seismic element viz. columns, frames, Walls & subsequently to the foundation finally to the soil.

Indian seismic code

* The codes ensure saftey of buildings under earthquake extitation - IS : 1893 - 1962

* Recommendations for earthquake resistant design of structure.

* IS 1893-1984, the country has divided into five zones in which one can reasonably forecast the intensity of earthquake shock which will occu in the event of future earthquake.

Design philosophy adopted for earthquake resistant structure :-

* The extreme loading condition caused by an earthquake and also the low probability of such an event occuring with in the expected life of a structure.

* The following dual design philosophy is usually adopted.

(i) The structure is designed to resist the expected intensity of ground motion due to a moderate earthquake, so that no significant damage is caused to the basic structure.

(ii) The structure should also able to withstand & resist total collapse in the unlikely event of a severe earthquake occuring during its life time.

- The designer is economically justified this case to allow some marginal damag but total collapse and loss of time must be avoided.

Additive shear :-

- It will be super-imposed for a statically applied eccentricity of ± 0.05 b; with respect to centre of rigidity.

Seismic dampers :-

- Dampers refers to any process that causes an oscillation in a system to decay rapidly to zero amplitude.
- It is a very important Pheromenon in vibration supressior or isolation.
- Damping causes the energy to be diverted from vibration to other energy sinks.

Types of dampers :-

- Metallic dampers (or) yielding dampers
- Friction dampers
- Viscous dampers

Metallic dampers :-

- Metallic dampers are made up of steel.
- The energy is absorbed by metallic components that yield.
- They are designed to deform so much when the building vibrates during an earthquake, which they cannot return to their original shape.
- This Permanent deformation is called inelastic deformation.

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- It uses some of the earthquake energy, which goes into building.

- <u>Types of metallic dampers</u>
  - (i) X-shaped plate dampers → two braces meet
  - (ii) Pulling & pushing dampers → sideways & making it deform.

- As the building vibrates, the braces stretch & compress.

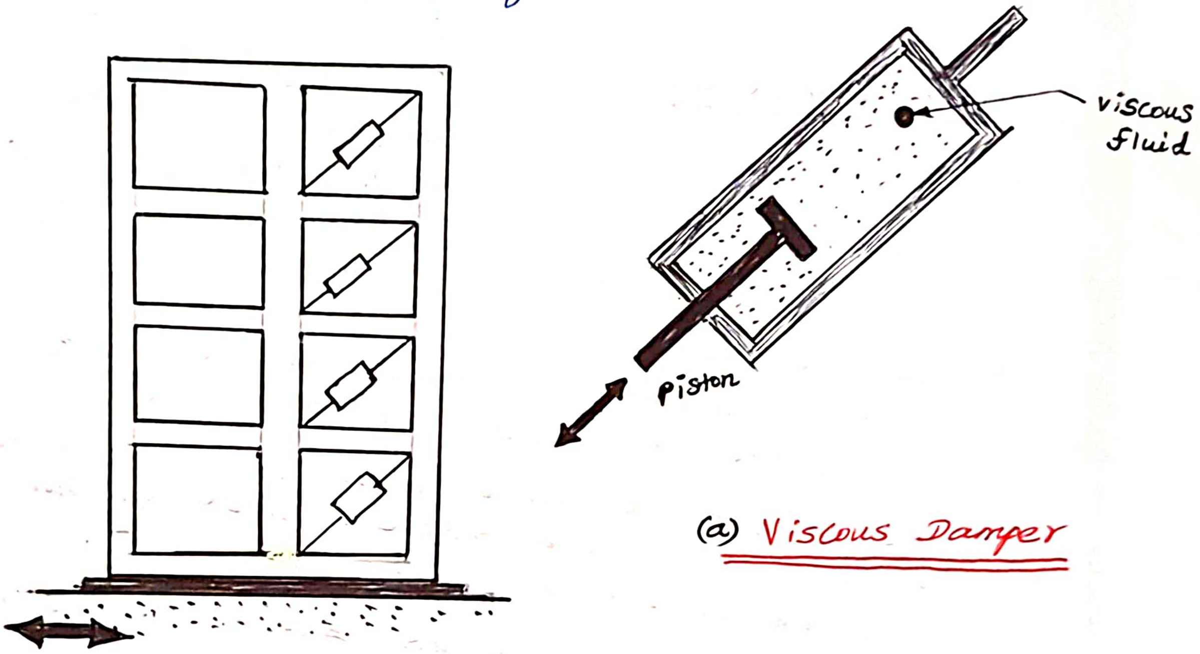
Friction dampers :-

- Friction dampers are designed to have moving parts that will slide over each other during a strong earthquake.
- When the parts slide over each other, they create friction, which uses some of the energy from the earthquake that goes into the building.
- The damper is made up ~~off~~ from a set of steel plates, which have slotted holes in them, and they are bolted together.
- At high enough forces, the plates can slide over each other creating friction.
- The plates are specially treated to increase the friction between them.

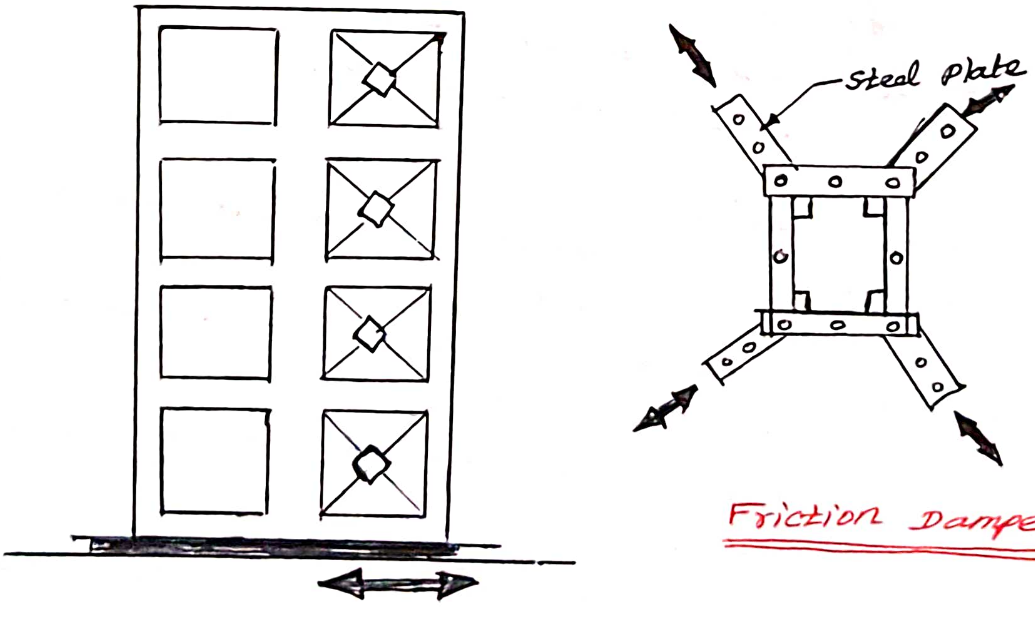
Viscous dampers :-

- viscous fluid dampers are similar to shock absorbers in a car.
- They consist of a closed cylinder containing a viscous fluid like oil.
- A piston rod is connected to a piston head with small holes in it.
- The piston can move in and out of the cylinder.
- It does The oil is forced to flow through holes in the piston head causing friction.

- * When the damper is Installed in a building. th friction converts some of the earthquake energy into the moving building into heat energy.
- * The damper is usually installed as part of a building's bracing system using diagonals.
- * As the building sways to and fro, the piston is for in and out of the cylinder.



(a) Viscous Damper



Friction Damper

IS code using Questions & Answers

IS : 1893 (Part I) - 2002 ; IS : 4326 : 1993 ; IS:13920:1993

1). Write short notes on special confining reinforcement an irregular building?

IS : 13920 - 1993 — clause : 7.4 — Page NO : 6 — special confining reinforcement

IS : 1893 (Pt 1) - 2002 — clause : 7.1 — Page NO : 17 to 22 — Irregular building
Table : 4 & 5 & diagram

2) Explain the load combination of earthquake design of structure?

IS : 1893 (Pt : 1) - 2002 — clause : 6.3 to 6.3.5.2 — Load combinations
P. NO : 13 to 14

3) Write short notes on response spectrum and Design spectrum.

Response spectrum → IS : 1893 (Pt : 1) - 2002 — clause : 7.8.4 & Notes
P.NO : 25 & 16 diagram

Design spectrum → IS : 1893 (Pt : 1) - 2002 — clause : 6.4 — Page NO : 14 to 16

4) In what manner behaviour of soft storey construction likely to be differ from regular construction in the event of earthquake?

IS : 1893 (Pt 1) - 2002 — clause : 7.10 to 7.11.1 — page NO : 27

5) Explain the general principle of IS : 1893 (Part 1) 2002?

IS : 1893 (Pt : 1) - 2002 — Page NO : 7 — Cl : 1.1 — scope
Page NO : 8 — ~~3~~ 7 or 8 — terminology

scope & terminology (7 or 8 topics)

6. Describe the codal based procedure for design of lateral force?

IS:1893(Pt: 1)-2002 – Page NO: 24 – 7.5 to 7.7

7. Explain P–Δ effect.

terminology.

IS: 1893 (Pt: 1)-2002 – page NO: 10 , Cl : 4.18

8) Define soil interaction characteristic?

IS: 1893 (Pt.1)-2002 – Page NO : 12 ; Cl : 6.1.4

9. What is storey drift?

Terminology

IS:1893 (Pt:1)-2002 – Page NO: 10 – Cl: 4.23
P.NO: 27 – Cl: 7.11.1

10) Explain soft storey? (or) open storey.

IS: 1893 (Pt: 1) – 2002 – Page.NO:10 – Cl: 4.20
Page NO: 27 – Cl : 7.10
P.NO: 18 – Table : 5
P.NO: 21
22 – Fig: 4

11) Discuss briefly the various codal provision for the dynamic analysis of a building?

* The earthquake resistance design of the structure is mainly to avoid loss of life & property.

* IS: 1893 (Pt: 1) – 2002 – First indian seismic code for all (different) type of structure.

Part 1 → deals with general criteria & building.
Part 2 → deals with liquid retaining structure
Part 3 → deals with bridges & retaining wall

* In this code take into an account of the design spectrum and response spectrum method.

Response spectrum method

(i) Horizontal seismic co-efficient $A_h = \frac{Z I}{2R}\left(\frac{S_a}{g}\right)$

IS:1893-Pt(1) – 2002 ; P.NO: ; Cl:

Explain in detail about detailing as per IS : 13920 - 1993.

IS : 13920 - 1993 - P.NO : 5 Cl : 7.1

(same as April - May 2018, Q.NO : 15 (b)